# Fuzzy Ruin Probability on Continuous Risk Model with Interference

LIU Zhaojun
School of Mathematics and Information Science, Shandong Institute of Business & Technology, Yantai, P.R. China, 264005
watertrees2003@yahoo.com.cn

**Abstract:** A double compound Poisson process risk model with interference is introduced firstly, according to normal approximate of compound Poisson process, a continuous risk model has been gotten, we use this continuous risk model to study the fuzzy ruin probability of risk model, the practicality of the risk model has been strengthened.
**Keywords:** Surplus, Poisson Process, Fuzzy Ruin Probability

## 1. Introduction

The ruin probability research of risk model has made great progress (1, 2). As to insurance company which service is great heavy, according to normal approximation(3), insurance premium collection and claims have been popularized as continuous risk model, some conclusions about ruin probability have been obtained(4). We will regard bankruptcy concept as a fuzzy concept in our article, on the one hand, it means insurance company's surplus begins at one value to change from approximating bankruptcy to bankruptcy in quantity; on the other hand, it means there is risk at times, insurance company's surplus can not be big enough to escape bankruptcy risk. It's just that the bigger surplus the smaller membership degree to bankruptcy. Basing on (4), we use fuzzy event probability to calculate ruin probability, and get fuzzy ruin probability. It's more advantageous to insurance company's risk management.

## 2. The Main Model

The definition of a double compound Poisson process risk model with interference is

$$U(t) = u + \sum_{i=1}^{M(t)} X_i - \sum_{i=1}^{N(t)} Y_i + \delta W(t),\ S(t) = \sum_{i=1}^{M(t)} X_i - \sum_{i=1}^{N(t)} Y_i + \delta W(t), \tag{1}$$

Where $(\Omega, F, P)$ is a probability space given, $u > 0$ denote insurance company's original capital; random sequences of $\{X_i \ge 0, i = 1, 2, ...\}$ and $\{Y_i \ge 0, i = 1, 2, ...\}$ are i.i.d respectively, which denote respectively each time collecting insurance premium and claims occurring; $\{M(t), t \ge 0\}$ and $\{N(t), t \ge 0\}$ are Poisson process which intensities are respectively $\lambda$ and $\mu$, they denote respectively the number of insurance policy and the number of claims occurring up to time $t$, and $M(0) = 0, N(0) = 0$; $\delta > 0$ is a constant; $W(t)$ is a normal wiener process, and denote a indefinite income or loss; $\{M(t), t \ge 0\}$, $\{N(t), t \ge 0\}$, $\{W(t), t \ge 0\}$, $\{X_i, i \ge 1\}$ and $\{Y_i, i \ge 1\}$ are independent each other. Let $p_1 = EX, q_1 = EY$, $p_i$ and $q_i$ $(i \ge 2)$ are respectively $i$ step origin moment of $\{X_i, i \ge 1\}$ and $\{Y_i, i \ge 1\}$. For ensuring insurance company's stable management, suppose $E[S(t)] > 0$, that is $\lambda p_1 - \mu q_1 > 0$, by this, definite security loading coefficient $\theta = \frac{\lambda p_1}{\mu q_1} - 1 > 0$, thus $(1+\theta)\mu q_1 = \lambda p_1$. In fact, $\lambda \square \mu$.

According to normal approximate of compound Poisson process, we have the following lemma (4):

For $t > 0$, denote $S_t = \sum_{i=1}^{M(t)} X_i$, then

$$\lim_{\lambda\to\infty} P\{\frac{S_t-\lambda tp_1}{\sqrt{\lambda tp_2}}\le x\}=\Phi(x) \tag{2}$$

By this lemma, for $t>0$, when $\lambda$ and $\mu$ is full big, we have approximately

$$\sum_{i=1}^{M(t)} X_i \sim N(\lambda tp_1,\lambda tp_2)\ ,\ \sum_{i=1}^{N(t)} Y_i \sim N(\mu tq_1,\mu tq_2) \tag{3}$$

Therefore, when $\lambda$ and $\mu$ are big enough, distribution of $S(t)=\sum_{i=1}^{M(t)} X_i-\sum_{i=1}^{N(t)} Y_i+\delta W(t)$ is approximate $N(\lambda tp_1-\mu tq_1,\lambda tp_2+\mu tq_2+\delta^2 t)$, so distribution of surplus $U(t)$ is approximate

$$N(u+\lambda tp_1-\mu tq_1,\lambda tp_2+\mu tq_2+\delta^2 t)\ ,$$

For study conveniently, denote $a=u+\lambda tp_1-\mu tq_1$, $\sigma^2=\lambda tp_2+\mu tq_2+\delta^2 t$, then distribution of surplus $U(t)$ is approximate $N(a,\sigma^2)$.

According to the people's understanding about bankruptcy fuzzy concept, we propose two type membership functions about bankruptcy fuzzy concept.

$$A_1(x)=\begin{cases}1 & -\infty<x\le 0\\ 1-\dfrac{x}{c} & 0<x\le c\\ 0 & c<x<+\infty\end{cases} \tag{4}$$

Where $c>0$ is a parameter, it's the maximum value of surplus $U(t)$ which causes people bankruptcy feeling.

$$A_2(x)=\begin{cases}1 & -\infty<x\le 0\\ e^{-\frac{x^2}{b}} & 0<x<+\infty\end{cases} \tag{5}$$

Where $b>0$ is a parameter.

Using (4) and (5), we can respectively obtain the following approximate results of fuzzy ruin probability.

**Theorem2.1** For $t>0$, $A_1(x)=\begin{cases}1 & -\infty<x\le 0\\ 1-\dfrac{x}{c} & 0<x\le c\\ 0 & c<x<+\infty\end{cases}$, when $\lambda$ and $\mu$ are big enough, fuzzy ruin probability is approximate

$$\Psi_t(u)\approx(1-\frac{a}{c})\Phi(\frac{c-a}{\sigma})+\frac{a}{c}\Phi(-\frac{a}{\sigma})-\frac{\sigma}{\sqrt{2\pi}c}(e^{-\frac{a^2}{2\sigma^2}}-e^{-\frac{(c-a)^2}{2\sigma^2}})\ . \tag{6}$$

**Proof** As $\lambda$ and $\mu$ are big enough, the distribution of surplus $U(t)$ is approximate $N(a,\sigma^2)$, where $a=u+\lambda tp_1-\mu tq_1$, $\sigma^2=\lambda tp_2+\mu tq_2+\delta^2 t$, then fuzzy ruin probability $\Psi_t(u)$ can be calculated approximately as following

$$\Psi_t(u)\approx P\{A_1(x)\}=\int_{-\infty}^{+\infty}A_1(x)p(x)dx=\int_{-\infty}^{0}p(x)dx+\int_0^c(1-\frac{x}{c})p(x)dx$$

$$=\int_{-\infty}^{0}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-a)^2}{2\sigma^2}}dx+\int_0^c(1-\frac{x}{c})\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-a)^2}{2\sigma^2}}dx$$

$$=F(0)+\int_0^c \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-a)^2}{2\sigma^2}}dx-\frac{1}{c}\int_0^c x\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-a)^2}{2\sigma^2}}dx$$

$$=\Phi(-\frac{a}{\sigma})+F(c)-F(0)-\frac{1}{c}\int_0^c (x-a+a)\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-a)^2}{2\sigma^2}}dx$$

$$=\Phi(-\frac{a}{\sigma})+F(c)-F(0)-\frac{1}{c}\{\frac{-\sigma}{\sqrt{2\pi}}\int_0^c e^{-\frac{(x-a)^2}{2\sigma^2}}d(-\frac{(x-a)^2}{2\sigma^2})+a[F(c)-F(0)]\}$$

$$=\Phi(-\frac{a}{\sigma})+F(c)-F(0)-\frac{1}{c}\{\frac{\sigma}{\sqrt{2\pi}}(e^{-\frac{a^2}{2\sigma^2}}-e^{-\frac{(c-a)^2}{2\sigma^2}})+a[F(c)-F(0)]\}$$

$$=(1-\frac{a}{c})\Phi(\frac{c-a}{\sigma})+\frac{a}{c}\Phi(-\frac{a}{\sigma})-\frac{\sigma}{\sqrt{2\pi}c}(e^{-\frac{a^2}{2\sigma^2}}-e^{-\frac{(c-a)^2}{2\sigma^2}})$$

**Theorem2.2** For $t>0$ , $A_2(x)=\begin{cases}1 & -\infty<x\le 0\\ e^{-\frac{x^2}{b}} & 0<x<+\infty\end{cases}$ , when $\lambda$ and $\mu$ are big enough, fuzzy ruin probability is approximate

$$\Psi_t(u)\approx\Phi(-\frac{a}{\sigma})+\sqrt{\frac{b}{2\sigma^2+b}}\Phi(\frac{a\sqrt{b}}{\sigma\sqrt{2\sigma^2+b}})e^{-\frac{a^2}{2\sigma^2+b}}. \tag{7}$$

**Proof** As $\lambda$ and $\mu$ are big enough, the distribution of surplus $U(t)$ is approximate $N(a,\sigma^2)$ , where $a=u+\lambda tp_1-\mu tq_1$ , $\sigma^2=\lambda tp_2+\mu tq_2+\delta^2 t$ , then fuzzy ruin probability $\Psi_t(u)$ can be calculated approximately as following

$$\Psi_t(u)\approx P\{A_2(x)\}=\int_{-\infty}^{+\infty}A_2(x)p(x)dx=\int_{-\infty}^{0}p(x)dx+\int_0^{+\infty}e^{-\frac{x^2}{b}}p(x)dx$$

$$=F(0)+\int_0^{+\infty}e^{-\frac{x^2}{b}}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-a)^2}{2\sigma^2}}dx=\Phi(-\frac{a}{\sigma})+\frac{1}{\sqrt{2\pi}\sigma}\int_0^{+\infty}e^{-(\frac{x^2}{b}+\frac{(x-a)^2}{2\sigma^2})}dx$$

$$=\Phi(-\frac{a}{\sigma})+\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{a^2}{2\sigma^2}}\int_0^{+\infty}e^{-(\frac{2\sigma^2+b}{2b\sigma^2}x^2-\frac{a}{\sigma^2}x)}dx$$

$$=\Phi(-\frac{a}{\sigma})+\sqrt{\frac{b}{2\sigma^2+b}}\Phi(\frac{a\sqrt{b}}{\sigma\sqrt{2\sigma^2+b}})e^{-\frac{a^2}{2\sigma^2+b}}$$

## 3. Conclusion

The above Theorem2.1 and Theorem2.2 have given two type fuzzy ruin probability. The constant $c>0$ in Theorem2.1 can be called as psychology early warning value of bankruptcy, we can investigate the administrative personnel and experts who have practical work experience to obtain individual empirical value and average it to get constant $c$. As to the constant $b>0$ in Theorem2.2 , we can investigate the administrative personnel and experts who have practical work experience, and designate one surplus $U(t)$ value which everyone is interested in, everyone gives $U(t)$ 's membership degree to bankruptcy membership function, to get a average, then it is put in bankruptcy membership function expression, thus we can get parameter $b$ . Thus, we can use (6) and (7) formulas to calculate fuzzy ruin probability. No doubt, these two theorems enable us to have more comprehensive understanding about ruin theory and make the researches for ruin probability more fitting in with reality.

## References


[1]. Yang Shanchao, Ma Zhong and Tan Jiyang. Risk Model of Random Premium Income. Mathematics in Economics, 2004, 21(1):1-5(in Chinese)

[2]. He Shuhong, Zhao Jine and Ma Lijuan. Ruin Probability in Risk Model with Double Compound Poisson Processes by Diffusion. Journal of Jishou University(Nature Science Edition), 2005, 26 (3):43-48(in Chinese)

[3]. Xie Zhigang and Han Tianxiong. Risk Theory and Non-life Insurance Calculation. Nan kai University Press, Tianjin, 2000:142-143(in Chinese)

[4]. Wang Yongmao, Liu Jiao and Guo Donglin. Continuous Risk Model with Interference. Mathematics in Economics, 2007, 24(1):27-30(in Chinese)