Coupled Numerical Simulation of Slope Stability with Unsaturated Soil under Rainfall Condition

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Abstract: Rainfall is one of main factors that affect the slope stability. Rainfall infiltration will change the degree of saturation determination of soil, and further influence the pore water pressure and the permeability coefficient of the medium. Based on the saturated-unsaturated seepage theory and the effect of rainfall infiltration, coupled with two phase medium of unsaturated soil and water, numerical simulation of soil slope under rainfall condition is carried out using the strength reduction finite element method. Changes of the slope displacement, pore pressure and safety factor under different rainfall periods and rainfall intensities are discussed. This study can provide important data for the slope stability analysis and landslide prediction.

Key words: unsaturated soil slope, rainfall infiltration, coupled analysis, strength reduction method, numerical simulation

1 Introduction

Rainfall, especially those lasting for a certain period, is one of main factors that affect the slope stability. Investigation data show that within the destructive factors, such as rainfall, excavating slope, underground mining at the lower part of the slop and large blasting, lasting rainfall will result in greater damage and landslide[1,2]. Water infiltration enhances the weight of soil, upslide impetus of slip surfaces and pore pressure, which threaten the slop stability apparently. The effective stress of soil is therefore reduced and the shear force of the soil decreases. On the other hand, after water immersion, the change of physical and mechanical properties of soil, the decreasing of strength, as well as the reduction of safety factor will lead to the instability of the slope. Therefore the analysis of the slope stability under rainfall condition is very much needed.

2 Establishment of Coupled Numerical Model

Soil is a multi phase porous medium. Fluid seepage of unsaturated soil obeys Darcy’s Law. Solid-liquid is coupled with volumetric strain and pore pressure. Coupled mathematical equation of saturated-unsaturated seepage can be obtained from fluid seepage theory of multi phase porous medium[3,4]:

\[ \sigma'_{ij,j} + (sp\delta_{ij})_j + f_i = 0 \]  \hspace{1cm} (1)

\[ s \dot{\varepsilon}_{kk} + V^F_{kk}k - cp = 0 \]  \hspace{1cm} (2)

where \( \sigma'_{ij} \) is effective stress, \( \delta_{ij} \) is Kronecker delta, \( p \) stands for pore pressure, \( f_i \) is body force, \( S \) is saturation, and \( \varepsilon_{kk} \) is body strain. The symbol \( (\cdot)_j \) denotes the derivation with respect to the coordinates \( X_j \), while \( (\cdot) \) indicates the derivation of time \( t \), \( V^F_{kk} \) is Darcy’s velocity. The packing coefficient \( c \) is given by

\[ c = \frac{ns}{k^F} \frac{ds}{dp} \]  \hspace{1cm} (3)
where \( n \) is porosity, \( k^w \) stands for bulk modulus of water, \( S \) is the function of pore pressure which is calculated using following formula:\[^5\] :

\[
s = s_r + \left[ \frac{1-s_r}{1+(\alpha P_r)^2} \right]^{1/2} \quad (p > 0) \]

\[
s = 1 \quad (p \geq 0) \]

where \( s_r \) is the minimum residual saturation rate, and \( \alpha \) is a parameter.

The velocity in the seepage field satisfies the Darcy law:

\[
\nu_j = k_{ij} \left[ -\frac{p}{r_F} + z \right] \]

where \( k_{ij} \) is Darcy permeability coefficient, \( r_F \) is the bulk density of water.

The calculation formula of permeability coefficient which changes with the variety of the pore pressure is given as follows:\[^5\] :

\[
k_{ij} = k_r (s) k_{ij}^* \]

\[
k_r = \left( \frac{s-s_r}{1-s_r} \right)^3 = \frac{1}{\left[ 1+(\alpha P_r)^2 \right]^{3/2}} \]

where \( k_{ij}^* \) is the permeability coefficient with complete saturation.

There are two kinds of definite conditions in seepage mechanics calculations. One is the boundary of constant water head, which could be determined by the water level of the observation hole. The other is the boundary of rainfall, which could be determined by the amount of rainfall infiltration\[^6\]. Based on the boundary of force and displacement along with Galerkin finite element method, coupling equation of the seepage and stress could be discretized.

3 Calculation Principle of Finite Element Strength Reduction Method

3.1 Principle of strength reduction method

Duncan points out that slope safety factor is the reduction degree of soil shear strength when slope reaches its critical state. Strength parameter of slope and sliding surface is reduced step by step when calculating, till it reaches its failure\[^7\], namely,

\[
c_F = c / F_s \]

\[
\phi_F = \tan^{-1} \left( \tan \left( \phi \right) / F_s \right) \]

where \( \phi \) is the internal frictional angle and \( c \) is cohesion of soil.

The static calculation of elastic-plastic finite element method is close to elastic problem at start. With increasing of \( F_s \) and decreasing of strength index, shear strength of slope soil decrease in finite element calculation, and the slope is then calculated circularly. Parts elements yield firstly and stress redistribution within elements. Local buckling occurs and the whole slope becomes unstable. The discount coefficient in which the the whole slope unstability occurs, namely the ratio of the actual shear strength index to damage index of suppositional failure, is the safety factor of the slope.

Compared with a limit equilibrium method, a finite element strength reduction method, which does not require to set the position of sliding surface previously, could calculate the safety factor of slope...
stability, the stress and strain of slope element, and the failure region. Therefore, the position of sliding
surface could be traced.

The whole instability failure of the soil is that the slide or collapse occurs along the sliding surface
and the whole sliding surface reaches its limit equilibrium, and the whole slope can not bear any more.
Meanwhile, acute transformation of the stress and strain of the slope take place. The soil suddenly slides
along the sliding surface and results in collapse.

However, the criteria of the soil instability failure, for which there is still no unified understanding,
is described as the plastic strain or acute transformatin of displacemnt along sliding surface, the
transfixion of plastic region from the foot to the top of the slope, the non-convergence in finite element
calculation with strength reduction method, etc. There are not great differences among other criteria[9,10].
Non-convergence of solution is selected in this study as the failure criteria. Non-convergence under the
appointed convergence criteria indicates that the stress distribution can not meet the failure criteria of
soil and mass-balance, which means the occurrence of failure.

3.2 Yield Criteria

Although Mohr-Coulomb yield criteria are widely used, yield surface in a three-dimensional space
is a pyramid surface with a hexagon section and brings difficulties to numerical calculation[11]. While,
Drucker-Prager yield criterion takes the influence of main stress and hydrostatic pressure to yield
strength into account, it suits to the soil properties very well and is used in many large finite element
softwares. The criterion is described as following:

\[ F = \alpha I_1 + J_2^{1/2} = k \]  (10)

where \( I_1 \) and \( J_2 \) are the first invariant of stress tensor and the second invariant of deviator stress
tensor, respectively. \( \alpha \) and \( k \) are parameters relate to internal frictional angle \( \phi \) and cohesion \( c \)
of soil, and are expressed as follows:

\[ \alpha = \frac{\sin \phi}{3}, \quad k = c \cos \phi \]  (11)

4 Example of Numerical Calculation

A section of slope is selected as the study object. Permeability coefficient is \( 2.78 \times 10^{-5} \) m/s. The
length of the slope calculation region is 160 meters and height is 20 meters. The total height of the
object is twice of the slope’s. Slope ratio is 1:2. The distance from the top of the slope to the left boundary
is 80 meters and 40 meters from the foot of the slope to the right boundary. The deformation modulus is
70MPa and poisson’s ratio is 0.3. Initial void ratio is 1.0 and the bulk density of soil is 18KN/m³.
Cohesion and internal frictional angle are 12 kPa and 20° respectively. DP yield criterion is selected with
the parameter of \( \alpha \) evaluated 2m\(^{-1}\) and minimum residual saturation rate \( s_r \) evaluation 0. The finite
element model of the object is shown in figure 1.

![Mesh of Finite Elements Model](image)

The boundary conditions used in calculation include: zero delivery as underside boundary, constant
water head of left and right side with the value of 32 and 16 meters respectively, the amount of rainfall
infiltration upside as the invariable rate flux-boundary. The amount of simulated rainfall is 10mm/h from
0 to 10 hours, 20mm/h from 10 to 20 hours, and 30mm/h from 20 to 30 hours.
Rainfall will not be considered initially during the calculation, that is steady seepage calculation. After the original pressure head is received, rainfall is involved in the computation. Slope displacement, pore pressure and safety factor could be gained at different time with given rainfall intensity condition.

As shown in displacement contours above, lateral displacement mainly appears at the slope surface. The maximal lateral displacement, with the value of 21mm, appears at the foot of the slope where the contours is denser. There is obvious change in lateral displacement from 5 to 35 hour. Displacement at the slope foot increases from 0.65 to 21 mm. Vertical displacement distributes widely and the maximal value appears at the top of the slope. Vertical displacement changes from -0.45mm to -15mm along with the time changing from 5 to 35 hours. One notable characteristic of lateral and vertical displacement is that gradient variation of displacement occurs directly at the superficial layer of slope and the visible slope is prone to superficial layer failure.
Rainfall infiltration results in the rise of pore pressure in unsaturated soil. With the increase in moisture content of soil and upgrade of saturated surface, pore pressure of slope changes from -120kPa to -80kPa and then -20kPa. Losing or reducing of matrix suction leads to the decrease in shear strength of soil. Stability of slope is then be affected.

Safety factor calculated with strength reduction method is 1.07 of 10 hour, 1.06 of 20 hour, 1.05 of 30 hour, and 1.02 of 40 hour. Slope is in a very dangerous state with such a weak safety factor. Safety factor mainly shows downtrend. While affected with the permeability coefficient, the change rate of safety factor does not adapt to the elevation of rainfall intensity. Calculations show that the maximal decrease in the safety factor does not stand at the beginning, but after a certain time of rainfall. In this case the rapidly decrease of safety factor appears after 40 hour’s rainfall.

5 Conclusions

(1) Based on the unsaturated seepage and rainfall infiltration, coupled with infiltration and stress, a strength reduction finite element method is used to analyze the changes of the slope displacement, pore pressure and safety factor. This study provides important data for landslide prediction and optimal design of slope.

(2) The maximal lateral and vertical displacement appear at the foot and the top of the slope respectively. One of notable characteristics in these two displacements is that the change region of displacement is mainly at the superficial layer, and not at the deep layer.

(3) The moisture content and pore pressure increase while matrix suction decreases or disappears under rainfall condition. The interaction of these two factors leads to the decreases in the shear strength of soil and slope stability.

(4) Safety factor, which decreases on the whole, is affected by the moisture content and infiltration parameter. The change rate of safety factor is not affected obviously by the elevation of rainfall intensity. The greatest decline rate of safety factor appears at 40 hour in this calculation. Therefore, the landslide induced by the rainfall infiltration does not occur at the beginning, but after a certain time of rainfall, which efficiently coincides with our observational results of engineering practice.

References:


