Analytical Solutions for Seepage Forces of Drained Circular Tunnel

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Abstract: A waterproof-type lining has to carry full hydrostatic pressure. However, a drainage-type usually partially responds to the hydrostatic pressure, and the hydrostatic pressure exerted on the lining is thus reduced. The seepage force, acting as a volume force upon the surrounding ground and the lining, was usually arbitrarily ignored but was taken into consideration in this paper. According to the theory of infinite aquifer well, the seepage force formulas of surrounding ground and lining under the conditions of axial symmetry are deduced. The impact of the permeability of lining on the surrounding ground seepage force cannot be ignored by comparing the previous seepage force formulas with the deduced formulas. Meanwhile, the impact of grouting on the seepage forces of the surrounding ground and lining is also considered in presented equations. For the formula \( f_{2m} \), the seepage force decreases with the increase of the n-value (n is the ratio of \( k_m/k_s \)), and when n>50, n-value has little effect on the surrounding ground seepage force.

Keywords: tunnel; surrounding ground; lining; seepage force; analytical solution

1 Introduction

According to the groundwater control methodology, underwater tunnels can be designed either as a waterproof- or drainage-type. The waterproof-type lining has to carry full hydrostatic pressure as well as the pressures generated by the surrounding ground. However, for a drainage-type, hydrostatic pressure acted upon the lining is reduced, and the seepage force, as a volume force upon the surrounding ground and the tunnel lining [1-4], should be taken into account in the lining design in a drained tunnel. Tunnel linings are hardly completely impermeable, as can be seen in a number of different publications and almost all tunnels in operation. Therefore, in many cases, Owner, Designer and Contractor tend to evaluate the benefits of considering partially or fully drained tunnel linings.

Usually two approaches, numerical and analytical methods, are used to estimate the seepage force acting upon the lining of drained tunnel[5]. In this paper, according to the theory of infinite aquifer well, we conducted the simple closed-form analytical solution for seepage force in a drained circular tunnel under the conditions of axial symmetry and steady-state groundwater flow, when considering the impact of the permeability of lining on the seepage forces of the surrounding ground. The difference in predictions of the seepage force is then investigated by comparing the previous seepage force formulas with the deduced formulas.

2 Review of the approximate solutions

Some researchers proposed approximate solutions for estimating the seepage force of the surrounding ground in homogeneous, isotropic, porous, and elastic medium, under the assumptions that the flow out of the tunnel is radial and Darcy’s law is valid. The approximate solutions for seepage force acted upon the lining can be described as follows.

To evaluate the seepage force of the surrounding ground in a drained tunnel, Bouvard and Pinto [6] presented the approximate solution which can be described as

\[
  f_B = -\frac{\gamma_s h}{r} \left( \frac{1}{\ln(R/r)} \right)
\]  

(1)
Where h is the vertical distance from the groundwater level to the tunnel depth center; \( \gamma_w \) is the unit weight of water; r is the tunnel radius and R is an arbitrary radial distance at which the seepage-induced excess pore-water pressure becomes nil.

Schleiss[7] recommended that the R-value was the vertical distance between the tunnel and the groundwater level, (i.e., R=h),

\[
f_s = -\frac{\gamma_w h}{r} \ln \left( \frac{h}{r} \right)
\]

Fernandez and Alvarez[8] used the image well method to deduce the steady-state groundwater flow in a circular tunnel

\[
f_r = -\frac{\gamma_w h}{r} \left[ \frac{-8h^2}{r^2} + \frac{4h}{r} \cos \theta \right]
\]

Where \( \theta \) is the angle measured clockwise from the crown of the tunnel.

Kyung-Ho Park et al.[5] used the conformal mapping to study the seepage forces acted upon the lining in a drained tunnel under the steady-state groundwater flow condition.

**Case 1:** zero water pressure along the tunnel circumference.

\[
f_{ik} = \gamma_w \frac{1 - 2\alpha \cos \theta' + \alpha^2}{2\alpha} \left[ 1 + \frac{H}{A} \ln \left( \frac{h}{r} + \frac{h^2}{r^2} - 1 \right) + \sum_{n=1}^{\infty} 2\alpha^n (1 + \alpha^{2n}) \cos n\theta' \right]
\]

**Case 2:** constant total head \( h_k \) along the tunnel circumference.

\[
f_{ik} = \gamma_w \frac{1 - 2\alpha \cos \theta' + \alpha^2}{2\alpha} \frac{H - h_k}{\ln \left( \frac{h}{r} + \frac{h^2}{r^2} - 1 \right)}
\]

Where \( \theta' \) is the angle measured anticlockwise from to the tunnel side wall; \( A = h\left(1 - \alpha^2\right)/(1 + \alpha^2) \); \( \alpha = (h - \sqrt{h^2 - r^2})/r \); H is the vertical distance between the tunnel center and the ground surface. If the groundwater table is below the ground surface, the solutions for eq. (4) and eq. (5) should be used with \( H=0 \) and \( h \) equal to the groundwater depth.

### 3 Present analytical solutions

**3.1 Calculation theory of seepage force**

To a drained tunnel, the seepage force acts as a volume force upon the surrounding ground and tunnel lining. The calculation theory toward the seepage force of the surrounding ground and lining can be obtained under the following simplified assumptions:

- homogeneous and isotropic permeability;
- both steady flow and Darcy’s law are available;
- cross section of circular tunnel is held at constant hydraulic potential.

With aforementioned simplified assumptions, three-dimensional flow around the tunnel can be described by the following Laplace equation

\[
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0
\]
Where $h$ is the total head, given by the sum of the pressure and elevation heads

$$h = z + \frac{p}{\gamma_w} + \frac{u^2}{2g}$$  \hspace{1cm} (7)

Where $p$ is the water pressure; $\gamma_w$ is the unit weight of water; $z$ is the elevation head; $g$ is the gravity acceleration; $u$ is the velocity of fluid medium. Usually, the velocity of fluid medium is comparatively small hence being ignored, so the eq. (7) can be simplified as

$$h = z + \frac{p}{\gamma_w}$$  \hspace{1cm} (8)

The gradient of the pore-water pressure $P$ namely is the seepage load, which is generated by the seepage field. The components of seepage load in the x, y, and z directions are

$$F_x = -\frac{\partial p}{\partial x} = -\gamma_w \frac{\partial h}{\partial x}$$
$$F_y = -\frac{\partial p}{\partial y} = -\gamma_w \frac{\partial h}{\partial y}$$
$$F_z = -\frac{\partial p}{\partial z} = -\gamma_w \frac{\partial h}{\partial z} + \gamma_w$$  \hspace{1cm} (9)

To an axisymmetric problem, eq. (9) can be written as

$$f_r = -\gamma_w \frac{\partial h}{\partial r}$$  \hspace{1cm} (10)

Where $f_r$ is the seepage force per unit volume in the radial direction.

### 3.2 Analytical solutions

According to the theory of infinite aquifer well and the preceding assumptions, we consider the tunnel profile and the boundary conditions as an axial symmetry circular (Fig.1), where $r_0$ is the inner radius of the lining; $r_1$ is the outer radius of the lining; $r_g$ is the outer radius of the grouting circle; $H$ is the far water head; $k_l$, $k_g$, and $k_m$ are the permeability coefficient of lining, grouting circle, and surrounding ground, respectively.

When a tunnel is excavated without a lining and a grouting circle, the flow out of the tunnel can be obtained by applying Darcy’s law ($Q / 2\pi r = k_u dh / dr$). Considering the boundary conditions ($r=r_i$, $h_1=0$; $r=H$, $h_1=H$), the inflow is
\[ Q = \frac{2\pi H k_m}{\ln \frac{H}{r_i}} \quad (11) \]

Again, using Darcy’s law, the water head \( h_1 \) in the surrounding ground without a lining and a grouting circle can be given
\[ h_1 = \frac{H}{\ln \frac{H}{r_i}} \quad (12) \]

Combining eq. (10) and eq. (12), the seepage force of the surrounding ground can be written as
\[ f_{1w} = -\frac{\gamma w H}{r \ln \frac{H}{r_i}} \quad (13) \]

After the lining and the grouting circle are constructed, the water head \( h_1 \) in the surrounding ground becomes \( h_2 \). According to Darcy’s law and Laplace equation, the inflow with a lining and a grouting circle can be obtained by the following equation
\[ Q = \frac{2\pi H k_m}{\ln \frac{H + k_m r_f + k_g r_g + k_s r_s}{r_g k_g r_i}} \quad (14) \]

Meanwhile, the water head in different area can be calculated by the following equations
\[ h_{2s} = \frac{H \ln \frac{r}{r_0}}{\ln \frac{r_0}{r_i} + \frac{k_s}{k_m} \ln \frac{r_g}{r_i} + \frac{k_s}{k_g} \ln \frac{r_s}{r_i}} \quad (r = r_0 \sim r_i) \quad (15-a) \]
\[ h_{2g} = H - \frac{H \left( \ln \frac{H}{r_g} + \frac{k_m}{k_g} \ln \frac{r_g}{r_i} \right)}{\ln \frac{H}{r_g} + \frac{k_m}{k_g} \ln \frac{r_g}{r_i} + \frac{k_m}{k_s} \ln \frac{r_s}{r_i}} \quad (r = r_i \sim r_g) \quad (15-b) \]
\[ h_{2m} = H - \frac{H \ln \frac{H}{r_g}}{\ln \frac{H}{r_g} + \frac{k_m}{k_g} \ln \frac{r_g}{r_i} + \frac{k_m}{k_s} \ln \frac{r_s}{r_i}} \quad (r = r_g \sim H) \quad (15-c) \]

Where \( h_{2s}, h_{2g}, \) and \( h_{2m} \) are the water head in the lining, grouting circle, and surrounding ground, respectively.

From eq. (10) and eq. (15), the seepage force of the different area can be written as
\[ f_{2s} = -\frac{\gamma w H}{r \left( \ln \frac{r_i}{r_0} + \frac{k_s}{k_m} \ln \frac{H}{r_g} + \frac{k_m}{k_g} \ln \frac{r_g}{r_i} \right)} \quad (r = r_0 \sim r_i) \quad (16-a) \]
\[ f_{2g} = \frac{-k_m \gamma_w H}{r \left( k_m \ln \frac{r_i}{r_0} + \ln \frac{H}{r_g} + k_m \ln \frac{r}{r_i} \right)} \quad (r = r_i \sim r_g) \quad (16-b) \]

\[ f_{2m} = \frac{-\gamma_s H}{r \left( k_m \ln \frac{r_i}{r_0} + \ln \frac{H}{r_g} + k_m \ln \frac{r}{r_i} \right)} \quad (r = r_g \sim H) \quad (16-c) \]

Where \( f_{2m}, f_{2p}, f_{2m} \) are the water head in lining, grouting circle, and surrounding ground, respectively.

When \( r = r_i \), and \( H = H_r \), the external water pressure acting upon the lining in a drained tunnel can be expressed as

\[ p = \frac{\gamma_w H \ln \frac{r}{r_0}}{\ln \frac{r_i}{r_0} + k_m \ln \frac{H}{r_g} + k_m \ln \frac{r}{r_i}} \quad (17) \]

Seepage force is a volume force, but in practice, we use usually the resultant force to calculate the deformation and force condition of the lining. The resultant force of the lining can be calculated by

\[ F = \int_{r_0}^{r_i} f_{2s} \, dr = \int_{r_0}^{r_i} \frac{-\gamma_s H}{r \left( \ln \frac{r_i}{r_0} + k_m \ln \frac{H}{r_g} + k_m \ln \frac{r}{r_i} \right)} \, dr = \frac{\gamma_s H \ln \frac{r_i}{r_0}}{\ln \frac{r_i}{r_0} + k_m \ln \frac{H}{r_g} + k_m \ln \frac{r}{r_i}} = p \quad (18) \]

According to the above eq. (17), we know that imperviously sealed tunnel (namely \( k_s = 0 \)) has to support full water pressure. If the grouting circle can prevent the water flowing into the tunnel, we also can conclude that the grouting circle would support the full water pressure.

### 4 Comparison of the solutions

In this research, the difference in seepage force predictions among the solutions \( f_S, f_F, f_{1K}, f_{2K}, f_{1m}, f_{2m} \) was investigated. Fig.2 show the results of seepage force with respect to \( r/h \) at the tunnel crown, side wall, and invert. The limits of \( r/h=0.001 \) and 0.999 are used, respectively, instead of \( r/h=0 \) and 1. In the equation \( f_{1K} \), we assume \( H = 0 \). In the equation \( f_{2K} \), we assume \( H = 0 \) and \( h_r = -h \). For the equation parameters in this paper in the deduced formulas \( f_{1m} \) and \( f_{2m} \), we assume \( k_m = 3.7 \times 10^{-5} \text{ cm/s}, k_s = 1 \times 10^{-6} \text{ cm/s}, \) the inner radius of lining \( r_0 = 3.0 \text{ m}, \) and the outer radius of the lining \( r_1 = 3.2 \text{ m}. \)
From Fig.2, we can see that the equations $f_S$ and $f_{1m}$ produce the similar results for seepage force at the tunnel crown, side wall, and invert, and when $r/h=0.3$, the seepage force is the smallest. The results are greater than those calculated by other approximate equations.

The formula $f_{2m}$ gives the smallest seepage force among all the equations. The greater the $r/h$-value, the smaller the value of seepage force. The main reason is that, in the $f_{2m}$ formula, we considered the impact of the permeability of lining on the surrounding ground seepage force, but compared to equation $f_{IK}$, the permeability of lining has little effect on the surrounding ground seepage force because the ratio of $k_m/k_s$ is 37.

At the tunnel crown, equations $f_{IK}$ and $f_{2m}$ produce the similar results for seepage force, but the former overestimate the seepage force. The solutions, $f_{IK}$, $f_F$ and $f_{2m}$ give the similar seepage force at the tunnel side wall and invert, equation $f_{2m}$ produce the smallest seepage force. Analysis of formula $f_{2m}$ is suitable for describing the surrounding ground seepage force, however, all other approximate equations overestimate the surrounding ground seepage force.

In order to investigate the impact of the permeability of lining on the surrounding ground seepage force, Fig.3 gives the surrounding ground seepage force by $f_{2m}$ formula. Fig.3 shows that when $n$ is equal to 1-namely the tunnel is excavated without a lining and a grouting circle, formula $f_{2m}$ gives the same result as formula $f_{1m}$. The seepage force decreases with the increase of the $n$-value, and when $n>50$, the $n$-value has little effect on the surrounding ground seepage force.

Fig.3 Surrounding ground seepage force by the formula $f_{2m}$ ($n$ is the ratio of $k_m/k_s$)

5 Conclusions

Simple closed-form analytical solutions of the surrounding ground seepage force and the lining in a drained circular tunnel under the steady-state groundwater flow condition have been deduced. From the above results and analyses, the following conclusions can be drawn:
(1) By reviewing the previous approximate solutions, the reported seepage formulas did not consider the impact of the permeability of lining to the surrounding ground seepage force. It is inconsistent with the practical engineering.

(2) In this paper, according to the theory of infinite aquifer well, the seepage force formulas of surrounding ground and lining under the conditions of axial symmetry are deduced. The impact of the permeability of lining, arbitrarily ignored by the documented seepage force formulas, on the surrounding ground seepage force is very large by comparing the reported formulas with the deduced formulas.

(3) Formula $f_{2m}$ is suitable for describing the surrounding ground seepage force, however, all other approximate equations overestimate the surrounding ground seepage force. For formula $f_{2m}$, the seepage force decreases with the increase of the n-value, and when $n>50$, the n-value has little effect on the surrounding ground seepage force.

Reference