On the Inventory-Transportation Integrated Optimization Problem

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Abstract: This paper assumes that the demand is a Poisson random process, the inventory strategy is (s, S), each demand amount Q* is same, scale of the economic transport quantity \( Q_0 = p Q^* \) (p is a positive integer), in which \( Q^* \) is the optimal order quantity of joint pricing used by the supplier and retailer, and the supplier will be entitled to ignore small group of temporary order bills to accumulate them to a certain amount and so as to distribute together, and the retailer also agrees to wait in allowed delaying time. Based on this hypothesis, we establish an inventory-transport integrated Optimization model, design an algorithm, and exemplify the rationality of the model and algorithm by a concrete example.

Keywords: Random Process; Inventory Strategy; Economic Order Quantity; Inventory-transport integrated optimization; Model

1 Introduction

Harris first published research paper on inventory-transportation integrated cost in 1913, and he proposed the classic EOQ (Economic Order Quantity) model[1]. In the model, products can be delivered as frequency of a row according to minimizing total cost. However, his research has not been gotten an attention in a long time due to economic conditions limitation in that time. Until the 1980s, his EOQ model was once again applied to the distribution system study. Burns et al.[2] is the earlier researchers in Inventory-Transportation Integrated Optimization (ITIO), consideration of their studies is without time windows, a single deliver & multi-demander system. On the assumption that the demand is identified, inventory maintenance costs of all customers is same, and transport cost is only associated with the distance, they studied respectively two kind of minimizing inventory-transportation costs distribution strategy, i.e., direct transportation (a vehicle to a customer) and break-bulk transportation (a vehicle to customers). Their analytical method, in essence, is to apply representative Aggregated Data rather than complicate data to find the solutions of practical problems on condition that the nature features of studied problems don’t be affected. This model is easier to be understood and solved, especially to be fit to solve large-scale logistics problem. On the assumption that the r customer’s demand is stochastic conditions, Federgruen and Zipkin (1984)[3] studied a resource inventory-transportation integrated optimization problem with one-to-many distribution network, proposed a nonlinear mixed-integer programming model, and applied Bender's Decomposition Approach to solve it. Ye Zhijian proposed a supplier inventory-replenishment-distribution strategy model under VMI model according to the characteristics of small demand quantities and multi-distribution times from customers, so as to count the highest inventory \( Q \) and distribution period \( T \) of the integration optimization and minimize the long-term average expectations total cost[4]. Yuan Qingda introduced a multi-path road distribution logistics system, expanded the existed study scope of the decision-making from the three levels of decision-making, i.e., strategic, tactical and operation, and constructed a descriptive mathematical model of such structural characteristics and an effective heuristics, and discussed the random demand of the inventory-transportation optimization problem. Assuming that the central inventory cost ordering cost needn’t be considered, the largest order quantity of retailer is a constant variable rather than decision-making variable, then replaced random demand for expectation greatest demand, therefore, transmitted random demand into certainty demand, and replenishment time of retailer in different group could be different[5]. Du Wen, Yuan Qingda, Zhou Zailing discussed a kind of random inventory-transportation integration optimization problem[6].

2 Inventory-Transportation Integrated Optimization Problem model
2.1 The Model Assumptions

Assumption 1: Demand is a Poisson random process. \( \left\{ X_n \right\}_{n=1,2, \cdots} \) is time intervals iid of random demand, and \( F(x) \) is its distribution function, and \( F(0)<1; \left\{ S_n \right\}_{n=1,2, \cdots} \) is time of the random demand showing, and is subject to \( k \)-Erlang distribution. So, demand process is \( N(t) = \sup \{ n : S_n \leq t \} \), demand accumulation process \( L(t) \) is accumulated demand quantity of all phase.

Assumption 2: The inventory strategy is \( (s, S) \), that is to say, supplier orders to the producer to renew inventory level to \( S \) when inventory levels lower than \( s \), at the same time, assuming that the supplier’s purchasing lead-time (or producer’s producing lead-time) can be neglected, that is, \( s = 0 \), then

\[
Z(t) = \begin{cases} 
  Q - I(t) & I(t) < Q_0 \\
  0 & I(t) \geq Q_0 
\end{cases}
\]

(2.1)

So the total times on distribution in a supplementary period are:

\[
i = \inf \left\{ i : iQ_0 > Q \right\} \quad \text{令: } \ i = \frac{Q}{Q_0}
\]

Assumption 3: The \( Q^* \) is same every time, and the bulk transportation economic quantity \( Q_0 = pQ^* \) (\( p \) is a positive integer), \( Q^* \) is optimal order quantity when suppliers and retailer joint pricing, then a retailer’s once random demand \( Q^* \) is a step, such as a times random service to customer arriving.

2.2 Model Parameters

Set \( A_r \) is inventory-replenishment every time, \( C_r \) is purchasing cost per unit, \( A_D \) is fixed cost of distribution every time, \( C_D \) is transportation cost per unit, \( h \) is inventory cost per unit per unit time, \( w \) is waiting cost of customer per goods unit per unit time.

2.3 Modeling

Knowing that expectation value of total cost in a replenishment cycle includes four parts: \( E_{SC} \), i.e., expectation value of inventory replenishment in every replenishment cycle; \( E_{HC} \), i.e., expectation value of inventory holding; \( E_{TC} \), transportation; \( E_{CWC} \), i.e., expectation value of customer waiting.

Set \( C(Q, Q_0) \) is the average expectation value of total cost in a replenishment cycle, then, by the refreshing theory, \( C(Q, Q_0) = E/E_L \), there, \( E \) is total cost of a replenishment cycle; \( E_L \) is a time length of a replenishment cycle. Once \( C(Q, Q_0) \) value is obtained, then we can model as following:

\[
\min C(Q, Q_0)
\]

s. t. \( Q \geq 0; Q_0 \geq 0 \)

Based on the refreshing theory, we research the costs of four parts. An expectation value of complete replenishment cycle \( E_L \) is

\[
E_L = E(T_i) = \frac{iQ_0}{\lambda}
\]

(2.2)
An expectation value $E_{SIC}$ of inventory replenishment in every replenishment cycle is:

$$E_{SIC} = A_r + C_{SIC}E_{OQ} = A_r + C_rQ_0^{'i}$$

(2.3)

When inventory level is $I(t)$,

$$I(t) = \begin{cases} 
Q(0 \leq t \leq T_i) \\
Q - Q_o(T_i < t \leq T_2) \\
\vdots \\
Q - (i'-1)Q_o(T_{i'-1} \leq t \leq T_i)
\end{cases}$$

(2.4)

An expectation value of inventory holding in every replenishment cycle is

$$E_{KIC} = hE \left( \int_0^T I(t) \, dt \right) = \left( \frac{i'Q_0}{\lambda}Q - \frac{i'(i'-1)}{2\lambda}Q_0^2 \right)h$$

(2.5)

An expectation value of transportation cost in every replenishment cycle is

$$E_{TC} = A_D i' + C_D i'Q_D$$

(2.6)

An expectation value of customer waiting cost in every replenishment cycle is

$$E_{CWC} = w i' \frac{Q_0(Q_0 - 1)}{2\lambda}$$

(2.7)

According to the above formula, it is available

$$C(Q, Q_D) = \frac{\lambda A_r}{Q} - \frac{h}{2}Q_0i' + \left( \frac{w}{2} + \frac{h}{2} \right)Q_0 + \frac{\lambda A_D}{Q_0} + hQ + \lambda C_R + \lambda C_D - \frac{w}{2}$$

3 Settling Model

3.1

Place $i' = \frac{Q}{Q_0}$ into above formula, then

$$C(Q, Q_D) = \frac{\lambda A_r}{Q} + \frac{h}{2}Q + \left( \frac{w}{2} + \frac{h}{2} \right)Q_0 + \frac{\lambda A_D}{Q_0} + \lambda C_R + \lambda C_D - \frac{w}{2}$$

Hessen Matrix determinant:

$$\begin{vmatrix}
\frac{\partial^2 C(Q, Q_D)}{\partial Q^2} & \frac{\partial^2 C(Q, Q_D)}{\partial Q \partial Q_0} & \frac{\partial^2 C(Q, Q_D)}{\partial Q_0^2} \\
\frac{\partial^2 C(Q, Q_D)}{\partial Q^2} & \frac{\partial^2 C(Q, Q_D)}{\partial Q \partial Q_0} & \frac{\partial^2 C(Q, Q_D)}{\partial Q_0^2} \\
\frac{\partial^2 C(Q, Q_D)}{\partial Q^2} & \frac{\partial^2 C(Q, Q_D)}{\partial Q \partial Q_0} & \frac{\partial^2 C(Q, Q_D)}{\partial Q_0^2}
\end{vmatrix} = \begin{vmatrix}
2 \lambda A_r & 0 \\
0 & 2 \lambda A_D \\
0 & 3 \lambda C_R
\end{vmatrix} = 4 \lambda^2 A_r A_D - 3 \lambda C_R > 0$$

So, $C(Q, p)$ is convex function, exists only most advantages which could be obtained from settling its first partial derivative, and its value is as follows:
\[
\begin{align*}
Q &= \sqrt{\frac{2 \lambda A_R}{h}} \\
Q_o &= \sqrt{\frac{2 \lambda A_D}{w + h}}
\end{align*}
\]

(3.1)

3.2 Computation Examples

Assuming that fixed costs for every time inventory replenishment \( A_R = 125 \), fixed costs for every time distribution \( A_D = 50 \), inventory costs per week for each unit \( h = 7 \); goods waiting costs of customers per week for each unit of goods \( w = 10 \); Poisson strength \( \lambda = 10 \) number units / per weeks; procurement costs of per unit \( C_R = 10 \); transport costs per unit \( C_D = 10 \). The computation results show in table 1. Clearly, Table 1 shows that inventory levels \( S = Q \) increases with \( A_R \) increasing accordingly, cumulative demand \( Q_0 \) increases with \( A_D \) increasing, \( Q, Q_0 \) will be reduced with keeping inventory costs \( h \) increasing, cumulative demand \( Q_0 \) should increase when customers waiting cost \( w \) increasing largely. It is easy to see that this model and the algorithm are reasonable.

<table>
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<tr>
<th>Data changes</th>
<th>( Q )</th>
<th>( Q_0 )</th>
<th>( i' )</th>
<th>( C(Q, Q_0) )</th>
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</table>

4 Conclusions

The theory in this paper and the optimization algorithms about specific logistics enterprises have some extent qualified analysis, and they could provide an initial quantitative data reference for enterprises logistics programming.

References
