Selection of Suppliers under Multi-product Purchase Based on Fuzzy Multi-objective Integer Program Model*

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Abstract: Supplier selection is increasingly seen as a strategic issue for enterprises, since supply performance can have a direct financial and operational impact on the business. And supplier selection decisions are complicated by the fact that various criteria must be considered in the decision-making process. In this paper, vagueness and imprecision of the objectives, constraints and parameters in this problem are considered, and a fuzzy multi-objective integer programming model is proposed to deal with the suppliers selection and order allocation problems in the supply chain system. Finally, a numerical example illustrates the method is feasible and effective.

Keywords: Suppliers, Multi-product, Order allocation, Supply chain, Fuzzy multi-objective integer program

1 Introduction

Suppliers selection is one of the most critical activities of purchasing management in a supply chain, because of the key role of suppliers’ performance on cost, quality and delivery in achieving the objectives of a supply chain. Suppliers selection is a multi-criteria decision-making (MCDM) problem which includes both tangible and intangible factors. The decision makers (DMs) always express their preferences on alternatives or on the criteria of suppliers, which can be used to help rank and split order quantities among suppliers for a variety of reasons including creating a constant environment of competitiveness. The preference information on alternatives of suppliers and on attributes belongs to the DMs’ subjective judgements. Generally, DMs’ subjective judgements are often uncertain and cannot be estimated by an exact numerical value. Thus, the problem of selecting suppliers has many uncertainties and becomes more difficult.

The suppliers selection problem has received considerable attention in academic research and literature, both in domestic and in international. Dickson thought quality was the most important criteria[1], whereas Weber and Current thought price was the highest-ranked factor, and they used a multi-objective approach to systematically analyze the trade-offs between conflicting criteria in suppliers selection problems[2-3]. Gaballa applied mixed integer programming to suppliers selection in a real case[4]. Rosenthal et al. developed a single mixed integer programming model for suppliers selection with bundling[5]. In this paper, a fuzzy multi-objective integer model under multi-product purchase is proposed for the suppliers selection problem. This fuzzy model enables the purchasing managers not only to consider the imprecision of information but also take the limitations of buyers and suppliers into account to calculate the order quantity assigned to each supplier. For example, Bellman and Zadeh suggested a fuzzy programming model for decision-making in fuzzy environments[6]. Zimmermann firstly used the Bellman and Zadeh method to solve fuzzy multi-objective linear programming problems[7]. In real cases, many input data are not known precisely for decision-making, the fuzzy multi-objective selection model can help the DMs to find out the appropriate order quantities to each supplier, and allows purchasing managers to manage supply chain performance on cost, quality, delivery and service, etc. Then, the proposed model is transformed into a fuzzy programming model and its equivalent crisp single-objective linear programming. This transformation reduces the dimension of the system, giving less computational complexity, and makes the application of fuzzy methodology more understandable.

*This work is supported by Soft Science Key Projects of Science and Technology of Heilongjiang Province under Grant No.GB07D204-1

40
2 Construction of the Fuzzy Multi-objective Integer Programming Model

2.1 Definitions and notation

Before presenting the multi-objective model for supplier selection, some definitions and notation can be shown as follows:

- \(i\) is the serial number of suppliers, \(i=1, 2, \ldots, n\),
- \(j\) is the serial number of different products, \(j=1, 2, \ldots, J\),
- \(l\) is the serial number of optimized objective, \(l=1, 2, \ldots, L\),
- \(r\) is the serial number of constrain, \(r=1, 2, \ldots, R\),
- \(x_{ij}\) is the quantity of the jth product purchased from the ith supplier,
- \(D_j\) is demand for the period of the jth product,
- \(p_{ij}\) is the purchase price without discount of the jth product purchased from the ith supplier,
- \(C_{ij}\) is capacity of the jth product purchased from the ith supplier,
- \(f_{ij}\) is the percentage of rejected quality level of the jth product purchased from the ith supplier,
- \(s_{ij}\) is the percentage of late delivery level of the jth product purchased from the ith supplier,
- \(h_{ij}\) is productivity quantity flexibility of the jth product purchased from the ith supplier,
- \(g_{ij}\) is the evaluation rank of the jth product purchased from the ith supplier,
- \(y_{ij}\), \(y_i\) is an variable 0 or 1,
- \(x_i\) is the quantity of product purchased from the ith supplier.

2.2 The multi-objective integer programming model

The multi-objective model for supplier selection problems under multi-product purchase is constructed with three objectives—cost, quality, and delivery, which can be shown as follows:

\[
\begin{align*}
\text{(1)} & \quad \min Z_1 = \sum_{i=1}^{n} \sum_{j=1}^{J} p_{ij} x_{ij}, \\
\text{(2)} & \quad \min Z_2 = \sum_{i=1}^{n} \sum_{j=1}^{J} f_{ij} x_{ij}, \\
\text{(3)} & \quad \min Z_3 = \sum_{i=1}^{n} \sum_{j=1}^{J} s_{ij} x_{ij},
\end{align*}
\]

\[
\begin{align*}
\text{(4)} & \quad \sum_{j=1}^{J} x_{ij} \geq D_i, \\
\text{(5)} & \quad x_{ij} \leq C_{ij}, \\
\text{(6)} & \quad \sum_{j=1}^{J} h_{ij} x_{ij} \geq D_i, \\
\text{(7)} & \quad \sum_{j=1}^{J} g_{ij} x_{ij} \geq D_i.
\end{align*}
\]

\[
y_{ij} = \begin{cases} 
0 & x_{ij} = 0 \\
1 & x_{ij} > 0 \end{cases}, \quad \text{and} \quad x_{ij} \times (y_j - 1) = 0, \\
y_i = \begin{cases} 
0 & x_i = 0 \\
1 & x_i > 0 \end{cases}, \quad \text{and} \quad x_i \times (y_i - 1) = 0.
\]

Where three objective functions (1)-(3) are formulated to minimize cost, rejected quality, and late delivery level of purchased items, respectively. Constraint (4) ensures that demand is satisfied. Constraint set (5) means that order quantity of each supplier should be equal or less than its capacity. Constraint set (6) represents productivity flexibility. Constraint set (7) represents evaluation rank. Constraint set (8) represents the buyer purchase the jth product from the ith supplier. Constraint set (9) represents the buyer purchase product from the ith supplier and constraint set (10) prohibits negative orders.

In a real case, DMs do not have exact and complete information related to decision objectives and constraints. For suppliers selection problems the collected data does not behave crisply and they are typically fuzzy in nature. A fuzzy multi-objective model is developed to deal with the problem.
2.3 The fuzzy multi-objective integer programming model

A general multi-objective model for the suppliers selection problem can be stated as follows:

\[
\min Z_1, Z_2, \ldots, Z_p, \quad (11) \quad \text{s.t.:} \quad x \in X_d, X_d = \left\{ x / g(x) = \sum_{i=1}^{n} a_{i} x_i \leq b_i, r = 1, 2, \ldots, m, x \geq 0 \right\}. \quad (12)
\]

Where the \( Z_1, Z_2, \ldots, Z_p \) are the negative objectives for minimization, such as cost, late delivery. \( X_d \) is the set of feasible solutions which satisfy the constraints, such as buyer demand, supplier capacity.

It was shown that a linear programming model for Problems (11) and (12) with fuzzy goals and fuzzy constraints may be presented as follows:

Find a vector \( x^T = (x_1, x_2, \ldots, x_n) \) to satisfy:

\[
\hat{Z}_d = \sum_{i=1}^{n} c_{i} x_i \leq \hat{b}_i, r = 1, 2, \ldots, h \quad \text{(for fuzzy constraints)} \quad (13)
\]

\[
g(x) = \sum_{i=1}^{n} a_{i} x_i \leq b_i, q = h+1, h+2, \ldots, m \quad \text{(for deterministic constraints)} \quad (14)
\]

\[
x_i \geq 0, i = 1, 2, \ldots, n, j = 1, 2, \ldots, J. \quad (15)
\]

Where \( c_{i} \), \( a_{i} \) and \( b_i \) are crisp or fuzzy values.

In this model, the sign “\( \leq \)” indicates the fuzzy environment. The symbol “\( \leq \)” in the constraints set denotes the fuzzyfied version of “\( \leq \)” and has linguistic interpretation “essentially smaller than or equal to” and the symbol “\( \geq \)” has linguistic interpretation “essentially grater than or equal to”. And \( Z_d^0 \) is the aspiration levels that the decision-maker wants to reach.

Therefore, the fuzzy multi-objective integer programming model of suppliers selection under multi-product purchase can be shown as follows:

\[
\sum_{i=1}^{n} \sum_{j=1}^{d} p_{ij} x_{ij} \leq Z_d^0, \quad (16)
\]

\[
\sum_{i=1}^{n} \sum_{j=1}^{d} f_{ij} x_{ij} \leq Z_r^0, \quad (17)
\]

\[
\sum_{i=1}^{n} \sum_{j=1}^{d} g_{ij} x_{ij} \leq Z_g^0, \quad (18)
\]

\[
\sum_{i=1}^{n} \sum_{j=1}^{d} h_{ij} x_{ij} \geq D_i, \quad (19)
\]

\[
x_{ij} \leq C_j, \quad (20)
\]

\[
x_{ij} \geq 0, \quad y_{ij} \geq 0, \quad \text{and} \quad x_{ij} \times (y_{ij} - 1) = 0, \quad (21)
\]

\[
0 \leq x_{ij} \leq 1, \quad x_{ij} \text{ is an integer variable,} \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, J. \quad (22)
\]

2.4 Determination of the membership function

In fuzzy programming modeling, a fuzzy solution is given by the intersection of all the fuzzy sets representing either fuzzy objectives or fuzzy constraints. The fuzzy solution for all fuzzy objectives and fuzzy constraints may be given as follows:

\[
\mu_{(x)} = \left\{ \bigcap_{i=1}^{n} \mu_{x_i} (x) \right\} \bigcap \left\{ \bigcap_{r=1}^{m} \mu_{x_r} (x) \right\} \quad (23)
\]

The optimal solution \( x^* \) is given as follows:

\[
\mu_{x^*} (x) = \max_{x \in X_d^*} \mu_{x} (x) = \max_{x \in X_d^*} \left[ \min_{i=1,2,\ldots,n} \mu_{x_i} (x), \min_{r=1,2,\ldots,m} \mu_{x_r} (x) \right] \quad (24)
\]
Zimmermann extended his fuzzy programming approach to the fuzzy multi-objective programming problem[8-9]. He expressed objective functions $Z_k, k = 1, 2, \cdots, p,$ and fuzzy constraints by fuzzy sets whose membership functions increase linearly from 0 to 1. And he solved problems (10)-(12) by using fuzzy linear programming. He formulated the fuzzy linear programming by separating every objective function $Z_i$ into its maximum $Z_i^+$ and minimum $Z_i^-$ value by solving:

$$Z_i^+ = \max Z_i, x \in X_i, Z_i^- = \min Z_i, x \in X_i, \quad (29)$$

Where $Z_i^+$ is the maximum value (worst solution) of negative objective $Z_i$, $Z_i^-$ is obtained through solving the multi-objective problem as a single objective using, each time, only one objective and $x \in X_i$ means that solutions must satisfy constraints while $X_i$ is the set of all optimal solutions through solving as single objective.

The membership function for minimization goals ($Z_k$) is given as follows:

$$\mu_k(x) = \begin{cases} 
1, & Z_i \leq Z_i^- \\
(Z_i^- - Z_k(x))/(Z_i^- - Z_i^+), & Z_i^- \leq Z_k(x) \leq Z_i^+, k = 1, 2, \cdots, p \\
0, & Z_i \geq Z_i^+. 
\end{cases} \quad (30)$$

The linear membership function for the fuzzy constraints is given as

$$\mu_r(x) = \begin{cases} 
1, & g_r(x) \leq b_r \\
1 - (g_r(x) - b_r)/d_r, & b_r \leq g_r(x) \leq b_r + d_r, r = 1, 2, \cdots, h \\
0, & g_r(x) \geq b_r + d_r. 
\end{cases} \quad (31)$$

Where $d_r$ is the subjectively chosen constants expressing the limit of the admissible violation of the $r$th inequalities constraints (tolerance interval). It is assumed that the $r$th membership function should be 1 if the $r$th constraint is well satisfied, and 0 if the $r$th constraint is violated beyond its limit $d_r$. It can be shown in Figure 1.

![Figure 1: Objective function as fuzzy number](attachment:image.png)

2.6 Solution of fuzzy model

In order to find optimal solution ($x^*$) in the above fuzzy model, it is equivalent to solving the following crisp model:

Maximize $\lambda \quad (32)$

s.t.: $\lambda \leq \mu_{z_k}(x), k = 1, 2, \cdots, p$ (for all objective functions) (33)

$\lambda \leq \mu_{z_r}(x), r = 1, 2, \cdots, h$ (for fuzzy constraints) (34)

$g_{r_p}(x) \leq b_{r_p}, p = h+1, h+2, \cdots, m$ (for deterministic constraints) (35)

$x_i \geq 0, i = 1, 2, \cdots, n$ and $\lambda \in [0,1] \quad (36)$
Where \( \mu_D(x) \), \( \mu_{O_i}(x) \) and \( \mu_{C_j}(x) \) represent the membership functions of solution, objective functions and constraints.

Then, the linear programming software LINDO/LINGO is used to solve this problem.

3 Numerical Example

In this section, an numerical example, which is designed and performed by concrete data, is given to testify the feasibility and effectiveness of the fuzzy multi-objective mixed integer programming model method of suppliers selection under multi-product purchases. For supplying four products to a market assume that five suppliers should be managed. The purchasing criteria are cost, quality and delivery. The capacity constraints of suppliers are also considered. It is assumed that the input data from suppliers' performance on these criteria are not known precisely. And the fuzzy value of suppliers' capacity is 10%.

The values of their cost, rejected quality and late delivery and constraints of suppliers are presented in Table 1. The demand is a fuzzy number, as shown in Table 2.

| Table 1  Suppliers information |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Product | Suppliers      | Purchase Price  | Rejected Quality| Late Delivery   | Productivity    | Evaluation Rank | Capacity |
| Type    |                |                |                 |                 |                 |                 |          |
| 1       | 1              | 2.5            | 0.03            | 0.10            | 0.04            | 0.94            | 1000     |
|         | 3              | 2.0            | 0.04            | 0.05            | 0.05            | 0.91            | 2000     |
| 2       | 1              | 5.0            | 0.03            | 0.16            | 0.02            | 0.89            | 3000     |
|         | 2              | 4.5            | 0.02            | 0.09            | 0.01            | 0.92            | 4000     |
| 3       | 4              | 1.0            | 0.03            | 0.10            | 0.02            | 0.94            | 4000     |
|         | 5              | 2.0            | 0.02            | 0.08            | 0.03            | 0.92            | 2000     |
|         | 3              | 2.5            | 0.04            | 0.02            | 0.04            | 0.94            | 2000     |
| 4       | 4              | 2.0            | 0.05            | 0.10            | 0.03            | 0.95            | 3000     |
|         | 5              | 3.0            | 0.06            | 0              | 0.02            | 0.94            | 2500     |

| Table 2  Purchaser Information |
|---------|-----------------|-----------------|-----------------|-----------------|
| Product | Productivity    | Flexibility     | Evaluation Rank | Demand |
| Type    |                  |                 |                 |      |
| 1       | 0.03            | 0.91            | 2500            |      |
| 2       | 0.01            | 0.86            | 6000            |      |
| 3       | 0.02            | 0.92            | 5000            |      |
| 4       | 0.02            | 0.90            | 4000            |      |

The multi-objective linear formulation of numerical example is presented as follows:

\[
\begin{align*}
\text{min } Z_1 &= 2.5x_{11} + 2.0x_{12} + 5.0x_{21} + 4.5x_{22} + 1.5x_{31} + 1.0x_{32} + 2.0x_{33} + 2.5x_{41} + 2.0x_{42} + 3.0x_{44} \\
\text{min } Z_2 &= 0.03x_{11} + 0.04x_{12} + 0.03x_{12} + 0.02x_{22} + 0.05x_{22} + 0.03x_{33} + 0.02x_{34} + 0.04x_{34} + 0.05x_{44} + 0.06x_{44} \\
\text{min } Z_3 &= 0.10x_{11} + 0.05x_{31} + 0.16x_{12} + 0.09x_{22} + 0.09x_{23} + 0.10x_{43} + 0.08x_{33} + 0.02x_{34} + 0.10x_{44} \\
\text{s.t.: } & x_{11} + x_{31} \geq 6000, x_{12} + x_{22} \geq 4000, x_{13} + x_{41} + x_{33} \geq 2500, x_{14} + x_{44} + x_{34} \geq 5000, \\
& x_{31} \leq 1000, x_{11} \leq 2000, x_{12} \leq 3000, x_{22} \leq 4000, x_{32} \leq 4000, x_{43} \leq 2500, x_{33} \leq 2000, x_{44} \leq 2000, \\
& x_{44} \leq 3000, x_{43} \leq 2500, 0.04x_{11} + 0.05x_{12} \geq 75, 0.02x_{12} + 0.01x_{22} \geq 60, 0.02x_{33} + 0.04x_{34} + 0.03x_{33} \geq 100, \\
& 0.04x_{34} + 0.03x_{44} + 0.02x_{44} \geq 80, 0.94x_{31} + 0.91x_{11} \geq 2275, 0.89x_{32} + 0.92x_{22} \geq 5160, \\
& 0.94x_{32} + 0.91x_{41} + 0.92x_{33} \geq 4600, 0.94x_{34} + 0.95x_{44} + 0.94x_{44} \geq 3600, \\
& x_{ij} \geq 0, \text{ and } x_{ij} \text{ is an integer variable, } i = 1, 2, \ldots, 4, \ j = 1, 2, \ldots, 5.
\end{align*}
\]

According to (29), the maximum and minimum value of the three objective functions can be calculated as follows:
Then, we have that the single objective programming model as follows:

Maximize $\lambda$

s.t. $\lambda \leq \frac{57687 - (2.5x_{11} + 2.0x_{12} + 5.0x_{13} + 4.5x_{21} + 1.5x_{31} + 1.0x_{41} + 2.0x_{14} + 2.5x_{44} + 2.0x_{44} + 3.0x_{44})}{5588.5}$

$\lambda \leq \frac{689.3 - (0.03x_{11} + 0.04x_{12} + 0.03x_{13} + 0.02x_{21} + 0.05x_{23} + 0.03x_{31} + 0.02x_{33} + 0.04x_{41} + 0.05x_{43} + 0.06x_{44})}{138.3}$

$\lambda \leq \frac{1845.9 - (0.10x_{11} + 0.05x_{12} + 0.16x_{13} + 0.09x_{21} + 0.09x_{23} + 0.10x_{31} + 0.08x_{33} + 0.02x_{41} + 0.10x_{43})}{568.9}$

$\lambda \leq \frac{1100 - x_{11}}{100}, \lambda \leq \frac{2200-x_{11}}{200}, \lambda \leq \frac{3300-x_{12}}{300}, \lambda \leq \frac{4400-x_{13}}{400}, \lambda \leq \frac{4400-x_{23}}{400}, \lambda \leq \frac{2750-x_{31}}{250}, \lambda \leq \frac{2200-x_{33}}{200}$.

The integer programming software LINDO/LINGO is used to solve this problem. The optimal solution for the above model is obtained as follows:

$\lambda_{max} = 0.673$, $x_{11} = 437$, $x_{12} = 2059$, $x_{13} = 1784$, $x_{22} = 4157$, $x_{33} = 1237$, $x_{41} = 2548$, $x_{53} = 1103$, $x_{44} = 2081$, $x_{44} = 1352, x_{44} = 535$.

From the result and the order quantity assigned to each supplier, we can find that purchasing quantity of some suppliers are more than their capacity, such as $x_{22}$ and $x_{34}$, which is caused by the vagueness and uncertainty of the information in this problem. Thus, the method enhances the accuracy of decision-making, and it’s also the merit of fuzzy multi-objective programming.

4 Conclusions

Suppliers selection is one of the most critical activities of purchasing management in a supply chain. In this paper, vagueness of input data are considered, and the fuzzy multi-objective mixed integer programming model is proposed to solve suppliers selection problem under multi-product purchases. This fuzzy model enables the purchasing managers not only to consider the imprecision of information but also take the capacity of purchasers and suppliers into account to calculate the order quantity assigned to each supplier. And the fuzzy objectives and fuzzy constraints are treated equivalently. However, in fact, in the suppliers selection problem, the fuzzy objectives and fuzzy constraints have unequal importance to DMs and other patterns. Thus, the weighted additive model, which different weights can be considered for various objectives and constraints, should be used to optimize this problem.

References


45