Study on an Improved Optimization Model of Logistics Distribution Vehicle Scheduling based on Stranger Problem

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Abstract This paper establishes an improved optimization model of logistics distribution vehicle scheduling based on “Stranger Problem”, which the common vehicle scheduling model does not consider, by defining two parameters named “Strange Degree” and “Strange Coefficient”, and makes a experimental computation by using improved C-W algorithm. The computational results prove the feasibility and efficiency of the improved optimization model.

Key Words Logistics Distribution, VSP, Stranger Problem, Strange Degree, Strange Coefficient

1 Introduction:
Nowadays, it is quite important for modern society to develop Logistics industry which is called “Acceleration Machine” for the development of economy. Distribution is an important part of logistics, and the cost of distribution, which is one of the most important parts in the costing of logistics, affects the whole benefit of logistics industry directly. However, most logistics enterprises in China have various weaknesses in distribution in present, such as the disequilibrium of transport job assignment, unreasonableness of distribution vehicle routing and serious wastes of transport resource. In proper statistical reports, the rate of empty load of transport vehicle is 37%, and the rate of empty load of logistics distribution vehicle is 39%, and the loss of efficient logistics distribution vehicle scheduling plan is one of the most important reasons to the situation.

The Optimization Problem of logistics distribution vehicle scheduling is always classified in Vehicle Routing Problem and Vehicle Scheduling Problem. Many experts and scholars have achieved fruitful studies in this area\textsuperscript{[1]}-\textsuperscript{[9]}. Zhang zhen (1995) improved a optimization method based a progressive approach\textsuperscript{[10]}; Li DaWei(1998) solved TSP by modifying the nearest distance searching heuristic, and proposed a new heuristic for the vehicle routing problem with time windows constraints\textsuperscript{[11]}; Li Jun, LANG Mao-xiang(2002) studied Optimization of Physical Distribution Routing Problem by Using Genetic Algorithm; Ma WeiMing(2003) studied VRPTW by online and competition game in his doctor paper.

In these conditions, several large logistics enterprise have applied VSP model and algorithm to their distribution practice. However, in the applying process the model is found that it is not suitable entirely to the practice, especially the “Stranger Problem” affected the efficiency of the VSP algorithm seriously. In order to solve “Stranger Problem”, this paper defines two parameters named “Strange Degree” and “Strange Coefficient”, establishes an improved optimization model of logistics distribution vehicle scheduling based on “Stranger Problem” and makes a experimental computation by using improved C-W algorithm.

2 Description of Current Model
The common description of VSP is: There are $k$ vehicles whose capability is $q$ stop in the same start spot (logistics centre) $v_0$, and the vehicles will offer cargoes to each of $l$ customers($v_1,\ldots,v_l$) whose demand is $g_i$ ($g_i \leq q$, $i=1,2,\ldots,l$).The coordinates of start spot and customers(expressed as $i(i=0,1,\ldots,l)$) are known. There is

$$y_{ki} = \begin{cases} 1, & k \text{ finish quest } i; \\ 0, & \text{ otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & k \text{ runs from } i \text{ to } j; \\ 0, & \text{ otherwise.} \end{cases}$$

And the VSP model is:
\[\begin{align*}
\min z &= \sum_i \sum_j \sum_k c_{ijk} x_{ijk} \\
\sum_k g_{ik} y_{ik} &\leq q, \forall k \\
\sum_k y_{ik} &= 1, i = 1, \ldots, l \\
\sum_j x_{ijk} &= y_{ik}, j = 0, \ldots, l; \forall k \\
\sum_j x_{ijk} &= 0, j = 0, \ldots, l; \forall k \\
X &= (x_{ijk}) \in S \\
x_{ik} &= 0 \text{or} 1, i = 0, 1, \ldots; \forall k \\
y_{ik} &= 0 \text{or} 1, i = 0, 1, \ldots; \forall k
\end{align*}\]

In model (1), \( c_{ij} \) expresses the transport cost from \( i \) to \( j \), and it means distance, expense, time, etc.

At present, the improvement of VSP mainly focuses on enhancing constraint such as time windows and rate of fully loaded. While the objective function is still the same as the common model: \( \min z = \sum_i \sum_j \sum_k c_{ijk} x_{ijk} \), it means that the objective is minimization of the total transport cost \( z \).

However, this objective function has a connotative strong constraint: To any vehicle \( k \), the transport cost from \( i \) to \( j \) is changeless. This means:

To any \( k \), if \( i \) and \( j \) are changeless, then \( c_{ij} \) is changeless. (2)

But in logistics practice, especially in urban distribution, the distribution area includes the whole city and surrounding towns, and the number of distribution customers always reaches thousands upon thousands. In this situation, it is impossible for the distribution vehicles to know all of the positions. “Strange Customer” always exists in every distribution task. Since common VSP model does not consider the “Strange Customer”, the optimization results of vehicle routing can not achieve prospective purpose constantly, and it is worse than original results in some occasions. As a result, it is necessary to describe “Strange Customer” in VSP model.

3 “Strange Problem” and the Improved Model
2.1 Strange Problem

“Strange customer” means the sets of strange customers of vehicles in distribution. There are two vehicles \( A \) and \( B \) to run a same distance from \( i \) to \( j \). \( A \) is familiar with \( i \) and \( j \), while \( B \) is unfamiliar to \( i \) and \( j \), so the time and expense \( A \) spends will be less than \( B \). This will lead to \( c_{ijA} \neq c_{ijB} \) (\( c \) means transport cost). As a result, to different vehicles, in the same distance, \( c_{ij} \) is not static. This problem means that the connotative constraint (2) is not logical, and affects the efficiency of VSP model. This is “Strange Problem”.

“Strange Problem” appears in logistics practice, especially in urban distribution often. In large logistics distribution, it is impossible for every vehicle to be familiar with each distribution area and each distribution customer. For example, the distribution area of a tobacco logistics enterprise includes the whole city and surrounding towns, and its number of distribution customers all round the area reaches thousands upon thousands. The vehicle routing is constant at past, and each vehicle has its own familiar customers. After optimization, since the “Strange Problem” is not considered, there are a mass of strange customers in the distribution tasks. Therefore, the vehicles waste a great deal of time on the way, and they can not finish the distribution tasks. In some occasions, the optimization results are worse than original results. From it, it can be concluded that “Strange Problem” badly affects the feasibility and efficiency of optimization of vehicle scheduling.
2.2 Two Definition

In order to solve “Stranger Problem”, “Strange Degree” \( q_{ki} \) is defined firstly. It means whether vehicle \( k \) is familiar with customer \( i \). The definition is (3).

Definition 1: “Strange Degree” \( q_{ki} \)

\[
q_{ki} = \begin{cases} 
1, & \text{if } k \text{ is not familiar with } i \\
0, & \text{otherwise} 
\end{cases}
\]  

Since common VSP model does not consider “Stranger Problem”, the transport cost is always \( c_{ij} \) to different \( k \). After considering “Stranger Problem”, the transport cost is not \( c_{ij} \) any more. The transport cost is classified to three classes:

If \( i \) and \( j \) are both familiar customers of \( k \), the transport cost is still \( c_{ij} \); If \( i \) or \( j \) is unfamiliar customer of \( k \), a coefficient \( \mu_i \) is added, and the transport cost changes to \( c_{ij} \mu_i \); If \( i \) and \( j \) are both unfamiliar customers of \( k \), a coefficient \( \mu_2 \) (\( \mu_2 \geq \mu_1 \)) is added, and the transport cost changes to \( c_{ij} \mu_1 \). \( \mu \) is a dependent variable of \( i, j, k \), and it is determined by \( q_{ki} \) and \( q_{kj} \), which are the Strange degrees. The definition is (4):

Definition 2: “Strange Coefficient” \( \mu_{ijk} \)

\[
\mu_{ijk} = \begin{cases} 
1, & q_{ki} = 0, q_{kj} = 0; \\
x_i, & q_{ki} = 0, q_{kj} = 1; q_{ki} = 1, q_{kj} = 0; \\
y_j, & q_{ki} = 1, q_{kj} = 1
\end{cases}
\]  

\( x_i \) = average transport cost of \( k \) from familiar to unfamiliar/average transport cost of \( k \) from familiar to familiar

\( y_j \) = average transport cost of \( k \) from unfamiliar to unfamiliar/average transport cost of \( k \) from familiar to familiar

2.3 Improvement of Model

Considering “Stranger Problem”, the common VSP model is improved as (5):

There are \( k \) some vehicles whose capability is \( q \) stop in the same start spot (logistics centre) \( v_0 \), and the vehicles will offer cargoes to each of \( l \) customers(\( v_i, \ldots, v_l \)) whose demand is \( g_i \) (\( g_i \leq q, i = 1, \ldots, l \)). The coordinates of start spot and customers (expressed as \( i(i = 0, 1, \ldots, l) \)) are known.

\[
\begin{align*}
\min z &= \sum_i \sum_j \sum_k c_{ij} x_{ijk} \mu_{ijk} \\
\sum_j g_i y_{ki} &\leq q, \forall k \\
\sum_i y_{ki} &= 1, i = 1, \ldots, l \\
\sum_j x_{ijk} &= y_{ki}, j = 0, \ldots, l; \forall k \\
\sum_j x_{ijk} &= y_{kj}, j = 0, \ldots, l; \forall k \\
X &= (x_{ijk}) \in S \\
x_{ijk} &= 0 \text{ or } 1, \ i = 0, 1, \ldots, l; \forall k \\
y_{ki} &= 0 \text{ or } 1, \ i = 0, 1, \ldots, l; \forall k
\end{align*}
\]  

(5)
In the model, $c_{ij}$ means common transport cost from $i$ to $j$, $\mu_{ik}$ means strange coefficient of $k$ from $i$ to $j$. Objective function is improved, and $c_{ij}x_{ijk}H_{ijk}$ is used to describe the transport cost of $k$ from $i$ to $j$.

4 Test of the Improved Model

C-W (Clarke-Wright) heuristic algorithm is widely used in calculation of VSP model. It is simple, easily understandable and agile. Many optimization algorithms entirely or partly apply C-W algorithm. C-W algorithm is improved based on the improved model, and validate the feasibility and efficiency of the model (5).

4.1 Algorithm Steps

Based on the ideas discussed above, the algorithm steps are designed below:

step0: Build customer set $L = \{1, 2, ..., L\}$,
vehicle set $K = \{k \mid k \in \{1, 2, 3, ..., \sum_i g_{ki}y_{ki} / q \}\}$, routing set $C = \emptyset$

step1: select $k$, while $\sum_i q_{ki} = \min\{\sum_i q_{ki}, k \in K, i \in L\}$;

step2: calculate $s(i, j, k)$, $i, j \in L$; let $M = \{s(i, j, k)\}$;

step3: arrange $s(i, j, k)$ from maximum to minimization in $M$;

step4: if $M = \emptyset$, then add routing set in calculation to $C$, go to step8; otherwise check $(i, j)$ to the first $s(i, j, k)$, if it satisfies one of conditions below:

(1) $i$ and $j$ are both not in the achieved routing;

(2) $i$ or $j$ is in the achieved routing, but they are both not the inner spots which are not link the start spot;

(3) $i$ and $j$ are in the different achieved routing, and are both not the inner spots, and one is the first spot while the other is the last spot.

then go to next step; otherwise go to step7;

step5: check total cargo $Q$ in routing after linking $i$ and $j$, if $Q \leq q$, then go to step6, otherwise go to step7;

step6: link $i$ and $j$, go to step7;

step7: let $M = M - s(i, j, k)$, go to step4;

step8: assign $k$ to $C_i$ in which there is least unfamiliar customers, and save the results, $K = K - \{k\}$, $L = L - C_i$, $C = \emptyset$, go to step9;

step9: if $L = \emptyset$, then finish; otherwise go to step1.

4.2 Explanation of Algorithm Steps

With the changes of the model, C-W heuristic algorithm is improved in order to solve the optimization problem of vehicle scheduling with “Stranger Problem”. The improvement mainly includes steps below:

step1: Consider the beginning calculation vehicle. The vehicle $k$ that has least strange customer ($\sum_i q_{ki}$) is chosen.

step2: Consider “Stranger Problem”, the Saving Cost will change. It is changed from $s(i, j)$
to \( s(i, j, k) \):

\[ c_{ij} \] means common transport cost from \( i \) to \( j \), and \( c_{ij} x_{ijk} \mu_{ijk} \) means transport cost of \( k \) from \( i \) to \( j \). Based C-W heuristic algorithm, the Saving Cost of \( k \) from \( i \) to \( j \) is:

\[
s(i, j, k) = c_{ij} \mu_{ijk} + c_{ij} \mu_{jik} - c_{ij} \mu_{ijk}
\]

step8: After each circle, there will be a routing set created. What routing is assigned to \( k \) should be considered then. The vehicle \( k \) that has least strange customer is chosen.

step9: Since “Strange Degree” of vehicles are different, \( s(i, j, k) \) is different to different \( k \). It means that the algorithm should be circled \[
\left[ \sum_i g_i y_{ki} / q \right]
\] times. After each circle, the customers who have been assigned to vehicles should be eliminated from customer set. The algorithm will end until the customer set \( L \) is empty.

4.3. Example

There are 8 transport tasks (numbered as 1, 2, …, 8). The Carriage of each task is \( g_i \) (unit is ton). The tasks are described in table 4-1. These tasks will be finished by the vehicles whose capacity is 8, and the vehicles are from the same distribution centre 0. The distance (unit is kilometer) between the distribution centre and each task spot is described in table 4-2.

<table>
<thead>
<tr>
<th>Taski</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_i ) (ton)</td>
<td>2</td>
<td>1.5</td>
<td>4.5</td>
<td>3</td>
<td>1.5</td>
<td>4</td>
<td>2.5</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4-2: Distance between each spot

<table>
<thead>
<tr>
<th>j ( i )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>40</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>200</td>
<td>100</td>
<td>160</td>
<td>80</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>0</td>
<td>65</td>
<td>40</td>
<td>100</td>
<td>50</td>
<td>75</td>
<td>110</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>65</td>
<td>0</td>
<td>75</td>
<td>100</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>40</td>
<td>75</td>
<td>0</td>
<td>100</td>
<td>50</td>
<td>90</td>
<td>90</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>75</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>50</td>
<td>100</td>
<td>50</td>
<td>100</td>
<td>0</td>
<td>70</td>
<td>90</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>75</td>
<td>75</td>
<td>90</td>
<td>75</td>
<td>70</td>
<td>0</td>
<td>70</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>160</td>
<td>110</td>
<td>75</td>
<td>90</td>
<td>75</td>
<td>90</td>
<td>70</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>100</td>
<td>75</td>
<td>150</td>
<td>100</td>
<td>75</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Here the transport cost is supposed in proportion to distance, and transport cost is expressed as distance. This means \( c_{ij} = d_{ij} \) \((i, j = 0,1, \ldots, 8)\). The question is how to arrange vehicles routing, and minimize the total transport cost.

1) At first calculate the number of the vehicles needed to delivery carriages \[
\left[ \sum_i g_i y_{ki} / q \right],
\] and the number is 3. Then choose the vehicle \( k \) based the table of strange degree (Table 4-3). Since vehicle 1 has least strangers, \( k = 1 \);
Table 4-3: Strange Degree of vehicles

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

2) Calculate the Saving Cost of every two spots when \( k = 1 \);

\[
\mu(i,j,1) = c_{0j} + c_{j1} - c_{i1},
\]

the “Strange Coefficient” is described in table 4-4.

Table 4-4: Strange Coefficient

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

For example, the saving cost between spot 1 and spot 7 is:

\[
\mu(1,7,1) = c_{17} + c_{71} - c_{11} = 40 + 160*2 - 110*2 = 140
\]

Other saving cost can be get in this way, and the data is arranged from maximum to minimization (table 4-5).

Table 4-5: Saving cost between spots when \( k=1 \)

<table>
<thead>
<tr>
<th></th>
<th>(1,7)</th>
<th>(5,7)</th>
<th>(6,7)</th>
<th>(4,7)</th>
<th>(5,6)</th>
<th>(2,7)</th>
<th>(3,7)</th>
<th>(3,5)</th>
<th>(3,7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s (1,7)</td>
<td>340</td>
<td>280</td>
<td>260</td>
<td>230</td>
<td>230</td>
<td>225</td>
<td>215</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>s (5,7)</td>
<td>340</td>
<td>280</td>
<td>260</td>
<td>230</td>
<td>230</td>
<td>225</td>
<td>215</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>s (6,7)</td>
<td>340</td>
<td>280</td>
<td>260</td>
<td>230</td>
<td>230</td>
<td>225</td>
<td>215</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>s (4,7)</td>
<td>340</td>
<td>280</td>
<td>260</td>
<td>230</td>
<td>230</td>
<td>225</td>
<td>215</td>
<td>115</td>
<td></td>
</tr>
</tbody>
</table>

3) Build routing under thinking of C-W Algorithm;

After check \( \mu(i,j,k) \) of each \( i \) and \( j \), the routing set is:

\[
0 \rightarrow 5 \rightarrow 7 \rightarrow 6 \rightarrow 0
\]

4) Choose the routing of vehicle 1;

Calculate the number of strangers of vehicle 1 in each routing. The number of routing 1 is 1, while the number of routing 2 and 3 is 0. Routing 2 is chosen in order. And the distribution routing of vehicle 1 is:

\[
0 \rightarrow 1 \rightarrow 3 \rightarrow 0
\]

5) Save the results after first circle, and begin the second circle;

Since routing \( 0 \rightarrow 1 \rightarrow 3 \rightarrow 0 \) and vehicle 1 has been selected, vehicle 1 and customer 1,3 should be eliminated from vehicle set and customer set. Then begin step 1.

6) When customer set is empty, calculation is ended. The final result is:

Vehicle 1: 0 \rightarrow 1 \rightarrow 3 \rightarrow 0

Vehicle 2: 0 \rightarrow 4 \rightarrow 2 \rightarrow 8 \rightarrow 0

Vehicle 3: 0 \rightarrow 5 \rightarrow 7 \rightarrow 6 \rightarrow 0
The calculation result of C-W algorithm based common VSP model is list as (6). The two results are compared in Table 3-6. The contrast shows that the result from improved model is better. It can be concluded that the improved model is feasible and solve the “Stranger Problem” efficiently.

\[
\begin{align*}
\text{Vehicle 1:} & \quad 0 \rightarrow 1 \rightarrow 3 \rightarrow 0 \\
\text{Vehicle 2:} & \quad 0 \rightarrow 4 \rightarrow 8 \rightarrow 2 \rightarrow 0 \\
\text{Vehicle 3:} & \quad 0 \rightarrow 7 \rightarrow 5 \rightarrow 6 \rightarrow 0 \\
\end{align*}
\] (6)

<table>
<thead>
<tr>
<th>Table 4-6: Contrast of two results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Based common VSP model</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>Cost of vehicle 1</td>
</tr>
<tr>
<td>Cost of vehicle 1</td>
</tr>
<tr>
<td>Cost of vehicle 1</td>
</tr>
<tr>
<td>Total Cost</td>
</tr>
</tbody>
</table>

5 Conclusion

(1) The “Stranger Problem” is inevitable in modern logistics distribution. Since the common VSP model does not consider the phenomenon, the result of optimization is not satisfied in practice.

(2) Considering “Stranger Problem” by defining two parameters named “Strange Degree” and “Strange Coefficient”, this paper improves optimization model of logistics distribution vehicle scheduling. Then the paper makes a experimental computation by using improved C-W algorithm. The computational results are better than the results based on the model of common VSP which does not consider “Stranger Problem”. As a result, the improved model is feasible and efficient.

(3) Using advanced theories and methods in logistics management is an important direction of development of modern logistics. Further more, people should think and study how to use the total functions of the excellent theories and methods in logistics practice.

Reference