A Differentiated Oligopoly Game Model of Initial Emission Permits Allocation

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Abstract The purpose of this paper is to investigate the allocation of output and the initial emission permits for two enterprises to produce differentiated product in an oligopoly game. By using a linear demand system, this paper obtains the allocation of output and the initial emission permits under the condition that the market power exists. Furthermore, it analyses the relation between the allocation results of output and the equilibrium price of emission permits. The study shows that when the equilibrium price of emission permits raises, the total equilibrium output will decrease, while the change rule of the individual output is complicated. If the cleanness parameter in the game model is same, the enterprises’ equilibrium output will decline equivalently. Assuming that the cleanness parameter in model is not same, and the other parameters in model satisfy different conditions, the market shares of output due to the change of the equilibrium price of emission permits transforms from polluting enterprise to cleaning one. It is needed to point out that with the raising of equilibrium price of emission permits, the individual output may increase on the condition that the related conditions hold.

Key words Oligopoly game, Initial emission permits allocation, Output allocation, Strategic interaction, Differentiated product

1 Introduction

Initial emission permits trading is an environment policy which can control pollution by using economic means and it has features of both environment quality assurance and cost-effective in the control of environment pollution. It is a more effective means by using the “invisible hand” of market to control environmental pollution. To some extent, although economists mainly concerned with fair and efficient problems, they have committed themselves to realizing initial emission permits trading in the process of the international political agenda. Now in the process of the implementation of environment policy, economists must deliberate on the initial allocation of emission right in the real world. Therefore, initial emission permits trading attract the universal concern of all countries in the world. In order to implement the system of the tradable initial emission permits, we must first solve a key problem that the allocation of initial emission permits from both a theoretical and a practical level. Taking many aspects together, we can conclude that there are mainly three modes about the initial allocation of emission permits, that is, the allocation of free, public auction, and a combination of two modes of allocation mentioned above. Up to the present, three former modes about the initial allocation of emission permits are discussed and studied in theoretical circles. Recently, with the unceasing implementations of the initial pollution transaction system’s in Western developed countries, for example, USA, Barde (1995) and Heller (1998) so on economist started to pay great attention to the initial emission permits allocation problem and carried on the discussion in this aspect. Long lists of arguments for the superiority of auctioning over other methods were presented by Cramton and Kerr (2002). By using a homogenous oligopoly game model, K. J. Sunnevåg (2003) studied the allocation of permits under two auction mechanisms, that is, the standard ascending auction and an alternative ascending-clock implementation of Vickrey-pricing. The purpose of this paper is to study the initial emission permits allocation problem for two enterprises to produce differentiated product in an oligopoly game.

2 The basic model

In this paper, we consider the allocation model of the initial emission permits of two firms
producing two differentiated goods. Let \( p_1 = A - Bq_1 - Dq_2 \) and \( p_2 = A - Bq_2 - Dq_1 \) be the linear inverse demand curves facing firm 1 and firm 2 respectively, where \( q_i \) \((i = 1, 2)\) denotes the output of firm \( i \) \((i = 1, 2)\); \( p_1 \) and \( p_2 \) denote the price of two different products produced by firm 1 and firm 2 respectively. Parameter \( A \) measures the market size or the reservation price, which is assumed to be equal across varieties for the sake of simplicity. As for parameters \( B \) and \( D \), we assume that \( 0 \leq D \leq B \). Notice that parameters \( D \) captures the degree of substitutability between the two different goods produced by two firms. In the limit case \( D = 0 \), goods are independent and each firm becomes a monopolist. In the opposite limit case \( D = B \), the goods produced by two firms are perfect substitutes and the model collapses into the homogeneous oligopoly model. Thus, the higher is parameter \( D \), the lower is the (symmetric) degree of differentiation.

In the process of production, we suppose that each unit produced generates emissions at the proportional rate of \( \delta_i \) \((i = 1, 2)\). However, each firm can substitute away from permits either by engaging in pollution abatement or reducing production, thus, the ultimate emission of firm \( i \) \((i = 1, 2)\) is \( q_i - d_i \), where \( d_i \) is firm \( i \)'s abatement level. Consequently, decisions in the product and permit markets are linked. Without loss of generality, we assume that the cost of abatement is assumed to be quadratic in both output and abatement per unit of output:

\[
k_i(d_i, q_i) = ad_i q_i + \beta(d_i q_i)^2, \quad i = 1, 2, \tag{1}
\]

where \( \alpha \) and \( \beta \) denote technological parameters \((\alpha, \beta \geq 0)\). Therefore, by the above formula with the assumption on \( \alpha \) and \( \beta \), we know that \( \partial k_i/ \partial q_i \geq 0, \ \partial k_i/ \partial d_i \geq 0 \) \((i = 1, 2)\), that is, the cost of abatement is non-decreasing function on \( \alpha \) and \( \beta \).

### 3 Duopoly game and initial emission permits allocation

Now we consider the above duopoly game model with the production of heterogeneous products. Tradable emission permits are considered, as an input with a fixed supply \( \overline{Q} \), which is exogenously determined by the authorities. Let \((Q_1, Q_2)\) be the initial emission permits of two firms such that \( Q_1 + Q_2 \). If both firms are assumed to be price takers in the permit transaction market, then firm \( i \)'s profit maximization problem becomes:

\[
\begin{align*}
\max_{q_i, d_i} \prod_i & = p_i q_i - c_i q_i - k_i (d_i, q_i) - \eta Q_i, \quad i = 1, 2, \\
Q_i &= \delta_i q_i - d_i q_i, \quad i = 1, 2.
\end{align*}
\tag{2}
\]

where \( \eta \) is the equilibrium price of permits. Substituting (1) into (2) and using the first order condition for profit maximization give the following results:

\[
\begin{align*}
\partial \prod_i / \partial d_i &= -\alpha q_i - 2 \beta d_i q_i^2 + \eta q_i \leq 0, \quad i = 1, 2, \\
d_i \times \partial \prod_i / \partial d_i &= d_i (-\alpha q_i - 2 \beta d_i q_i^2 + \eta q_i) = 0, \quad i = 1, 2, \\
d_i \geq 0, \quad i = 1, 2.
\end{align*}
\tag{3}
\]

Usually, we assume \( d_i > 0 \) \((i = 1, 2)\), thus from (3), we obtain \( d_i = (\eta - \alpha) / 2 \beta q_i \), \((i = 1, 2)\).

At the interior Cournot equilibrium, the first order condition of profit maximization for firm \( i \) is:

\[
\begin{align*}
\partial \prod_i / \partial q_i &= p_i - c_i + \partial p_i / \partial q_i - \eta (\delta_i - d_i) = 0, \quad i = 1, 2.
\end{align*}
\]
Therefore, $q_i = (A - c_i - Dq_j - \eta \delta_i) / 2B$, $i \neq j$, $i, j = 1, 2$, that is, firm $i$’s reaction function.

In addition, we assume that:

I. $A - c_i - \eta \delta_i > 0$, $i = 1, 2$;

II. $0 < D < \min\{A - c_1 - \eta \delta_1, A - c_2 - \eta \delta_2, 1\}$;

III. $0 < \delta_1 / 2\delta_2 < 1$, $0 < \delta_2 / 2\delta_1 < 1$. The assumption III implies that the difference between each unit produced generates emissions for two firms is not big.

Solving the system of linear equations composed by two firms’ reaction functions leads to the following Nash equilibrium quantity of two firms and the total production of two firms:

(4) $q_1^* = \frac{2B(A - c_i - \eta \delta_i) - D(A - c_2 - \eta \delta_2)}{4B^2 - D^2}$,

(5) $q_2^* = \frac{2B(A - c_2 - \eta \delta_2) - D(A - c_1 - \eta \delta_1)}{4B^2 - D^2}$.

By assumptions I and II, it is easy to check that the above Nash equilibrium quantity of two firms are interior solutions, that is, $q_1^* > 0$, $q_2^* > 0$. Taking the difference between (4) and (5) gives

$q_1^* - q_2^* = (2B + D)(c_1 + \eta \delta_1 - (c_2 + \eta \delta_2))$.

Obviously, the sign of $q_1^* - q_2^*$ depends on the sign of $(c_1 + \eta \delta_1) - (c_2 + \eta \delta_2)$: $q_1^* \geq q_2^*$ if and only if $(c_1 + \eta \delta_1) \geq (c_2 + \eta \delta_2)$; $q_1^* < q_2^*$ if and only if $(c_1 + \eta \delta_1) < (c_2 + \eta \delta_2)$.

To examine the effects of the equilibrium price of permits on the Nash equilibrium quantity of two firms, we differentiate (4) and (5) with respective to $\eta$. This yields:

$\frac{dq_1^*}{d\eta} = \frac{D\delta_2 - 2B\delta_1}{4B^2 - D^2}$; $\frac{dq_2^*}{d\eta} = \frac{D\delta_1 - 2B\delta_2}{4B^2 - D^2}$.

By (6) and assumption III, we know that $dq_i^* / d\eta < 0$ ($i = 1, 2$). This implies that an increase in $\eta$ decreases the Nash equilibrium quantity of two firms under satisfying assumption III. Furthermore, by use of (6), we obtain the following two results:

1. $\delta_1 = \delta_2 \Leftrightarrow dq_1^* / d\eta = dq_2^* / d\eta < 0$. It implies that an increase in $\eta$ decreases the same Nash equilibrium quantity range of two firms if and only if $\delta_1 = \delta_2$;

2. $\delta_1 > \delta_2 \Leftrightarrow dq_1^* / d\eta < dq_2^* / d\eta < 0$. It implies that under the condition that $\delta_1 > \delta_2$, we see a redistribution of production, where the less-polluting firm actually increases its production somewhat on behalf of the “dirty” firm. The inverse is also true.

Suppose that $\delta_1 >> \delta_2$, $\frac{\delta_2}{\delta_1} < \frac{D}{2B} < \min\{\frac{A - c_1 - \eta \delta_i}{A - c_2 - \eta \delta_2}, \frac{A - c_2 - \eta \delta_2}{A - c_1 - \eta \delta_1}, \frac{1}{2}\}$. Then we

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1 Here $\delta_1 >> \delta_2$ implies that $\delta_1$ is far more than $\delta_2$. Of course, in this case the assumption III does not hold.
Proposition 1. Under assumption I and the above two assumptions, the linear duopoly game model admits a unique interior Nash equilibrium quantity of two firms. The effect of parameter $\eta$ on the Nash equilibrium quantity of the “dirty” firm is negative; the effect of parameter $\eta$ on the Nash equilibrium quantity of the less-polluting firm is non-negative.

Proof. The conclusion that under assumption I and the above two assumptions, the linear duopoly game model admits a unique interior Nash equilibrium quantity of two firms is obviously. By use of the assumption that $\delta_2 / \delta_1 < D / 2B \leq 1/2 < \delta_1 / \delta_2$, we obtain the following:

$$\frac{\delta_2}{\delta_1} < D / 2B \leq 1/2 < \frac{\delta_1}{\delta_2}. \quad (7)$$

Hence by (6) and (7), we know that the following results hold:

$$\frac{dq_1}{d\eta} = \frac{D\delta_2 - 2B\delta_1}{4B^2 - D^2} < 0, \quad \frac{dq_2}{d\eta} = \frac{D\delta_1 - 2B\delta_2}{4B^2 - D^2} > 0.$$

This completes the proof.

Proposition 1 states that under some special restrictions, as permit price increases from zero, the Nash equilibrium quantity of the polluting firm will decrease, whereas the Nash equilibrium quantity of the less-polluting firm will increase.

![Figure 1](image_url)

Figure 1. The distribution of production between two firms (satisfying the conditions of Proposition 1).

Now we can allocate the available supply of permits $\overline{Q}$ on the basis of the above analysis. By the Nash equilibrium quantity of two firms, $Q_i = \delta_i q_i - d_i q_i$, and $d_i = (\eta - \alpha) / 2\beta q_i$, $i = 1, 2$, we obtain the initial emission permits allocation of two firms:

$$Q_i^* = \delta_i \frac{2B(A - c_1 - \eta\delta_i) - D(A - c_2 - \eta\delta_i)}{4B^2 - D^2} - d_i q_i^*$$

Figure 1 differs from the second part of Figure 1 of K. J. Sunnevåg (2003). As permit price increases, the equilibrium quantity point of Figure 1 in this paper transfers from $C(q_1^*, q_2^*)$ to $C'(q_1^{**}, q_2^{**})$ with $q_1^{**} < q_1^*$ and $q_2^{**} > q_2^*$, whereas the equilibrium quantity point of the second part of Figure 1 of K. J. Sunnevåg (2003) transfers from $B(q_1^*, q_2^*)$ to $B'(q_1^{**}, q_2^{**})$ with $q_1^{**} < q_1^*$ and $q_2^{**} < q_2^*$. 

931
\[
= \frac{2B(A - c_1 - \eta \delta_1) - D(A - c_2 - \eta \delta_2)}{4B^2 - D^2} - \frac{\eta - \alpha}{2\beta};
\]

\[Q_2^* = \bar{Q} - Q_1^*.\]

We give some numerical examples on the initial emission permits allocation. In these examples, the available supply of quotas is \(\bar{Q} = 1000\) permits, where each permit allows the firm to emit one unit of pollution. The other parameters are: \(A = 1500\), \(B = 0.3\), \(D = 0.25\), \(c_1 = c_2 = 200\); technological parameters \(\alpha = 100\), \(\beta = 0.4\).

**Case 1.** \(\delta_1 = 0.6\), \(\delta_2 = 0.4\)

Table 1. Distribution of production and emissions quota \((\delta_1 = 0.6\), \(\delta_2 = 0.4)\)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Firm 1</th>
<th>Firm 2</th>
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</thead>
<tbody>
<tr>
<td>(\eta = 425)</td>
<td>1157.98</td>
<td>288.54</td>
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<tr>
<td>(\eta = 335)</td>
<td>1236.64</td>
<td>448.23</td>
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<td>(\eta = 287)</td>
<td>1278.59</td>
<td>533.40</td>
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</table>

**Case 2.** \(\delta_1 = \delta_2 = 0.5\)

Table 2. Distribution of production and emissions quota \((\delta_1 = \delta_2 = 0.5)\)

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<td>(\eta = 425)</td>
<td>1279.41</td>
<td>233.46</td>
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<td>(\eta = 335)</td>
<td>1332.35</td>
<td>372.43</td>
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<td>(\eta = 287)</td>
<td>1360.59</td>
<td>446.55</td>
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</table>

**Case 3.** \(\delta_1 = 0.8\), \(\delta_2 = 0.2\)

Table 3. Distribution of production and emissions quota \((\delta_1 = 0.8\), \(\delta_2 = 0.2)\)

<table>
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<td>1023.53</td>
<td>506.32</td>
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<tr>
<td>(\eta = 335)</td>
<td>1045.21</td>
<td>542.42</td>
</tr>
<tr>
<td>(\eta = 287)</td>
<td>1114.59</td>
<td>657.92</td>
</tr>
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4 Conclusion

In this paper, the allocation problem of output and the initial emission permits for two firms to produce differentiated product in an oligopoly game is studied. On the basis of a linear demand system, the allocation result of output and the initial emission permits under the condition that the market power exists is obtained. Furthermore, the relation between the allocation results of output and the equilibrium price of emission permits is analyzed. The study shows that as the equilibrium price of emission permits increases, the total equilibrium output will decrease, whereas the change rule of the individual output is complicated. If the cleanness parameter in the game model is same, the firms’ equilibrium output will decline equivalently. Assuming that the cleanness parameter in model is not same, and the other parameters in model satisfy different conditions, the market shares of output due to the change of the equilibrium price of emission permits transforms from polluting enterprise to cleaning one. With the raising of equilibrium price of emission permits, the individual output may increase on the condition that the related conditions hold. Finally, in order to coincide with theoretical analysis, three numerical examples about the distribution of production and emissions quota are given.

References