Comparative Research on Stock Volatility between Shanghai Composite Index and Dow Jones

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Abstract: Accommodating exchange rate factors as exogenous disturbance, this paper proposes a mixed GARCH-Jump model to compares in general the volatility properties of returns series of the Shanghai composite index with those of the Dow Jones index. It also incorporates the asymmetry, clustering and leptokurtosis and fat-tail properties of returns volatility into an integrated analytic frame of so-called diffusion-jump. The fitness test on the model indicates that mixed GARCH-Jump model predicts the abnormality in the financial returns series of the emerging markets more powerfully than the single models.

Key words: GARCH-Jump model, Maximum likelihood estimation, Diffusion effect, Jump effect

1 Introduction

According to evidences form various empirical research, we can see that the returns based on the indexes of the emerging Asian stock markets have different characteristics from the western mature markets. A study of Bekaer and Harvey’s (2002) shows that more significant volatility, fat-tail and more strong forecastability are observed in Asian markets than in western markets. Aggarwal, Inclan and Leal (1999) indicated that the return rate of the emerging markets are mainly driven by policies and innovations, and the skewness of most of them are positive compared with negative ones in the western markets. We can expect an most intriguing area through deep research into these interesting properties in our country’s market.

Duan, Ritchken and Sun (2004) proposed non-linear jump-GARCH model to explore the correlationship between the jump of returns and that of the volatilities. Maheu and McCurdy (2004) introduced the time-varying variance and volatility clustering into the jump distribution function by using mixed jump-GARCH model. Aiming at the Southeastern Asian financial crises, Daal, Nake and Yu (2005) combined the above two models to study the impact of macro-disturbance on the returns of the Asian markets before and after the southeastern Asian financial crises.

Recently, scholars in China began to introduce the jump disturbance model. Xie Ci, DunYiyin (2003) modeled the heterogeneous jump diffusion process in our country’s interest rate, and they studied the non-linear dynamic properties of our country’s interest rate. LinHai, Zhen Zhenlong (2003) analyzed the applications of the GARCH in our country’s interest rate policies. Huang Boyi, Qiu Zhexiu (2002) who come from TaiWan also used jump model to analyze the volatility of short-term interest rate. Chen Min, Wu Guofu(2003a, 2003b)used the ACD-GARCH in which the Jump-diffusion is the core to analyze the non-linear volatility of the return of our stock market.

As is known to all, we have changeable policies and abnormal volatilities in our security market. Either our external conditions including institutional environments, quality of listed corporations and scale of finance or our internal conditions including essences of the investors is far from those of a mature market. Aiming at the characteristics, this paper models the volatility of returns of our country’s markets to analyze the non-linear properties of our country’s market.

2. Econometric Model

Suppose that the behavior of stock price is a diffusion process, so that the logarithm of the price ratio-\( r_t = \ln(P_t/P_{t-1}) \) follows a normal process. That is as follows:
\[
dP_t / P_{t-1} = adt + \delta dz_t
\]
\[
r_t = \ln(P_t / P_{t-1}) \sim N(\mu, \delta^2)
\]
\[
\mu = \alpha - \beta / 2
\]
(2.1) (2.2) (2.3)

Under the condition of discrete time, assume the stochastic item of error is a normalized normal process, then:
\[
dP_t / P_{t-1} = adt + \delta dz_t + dq_t
\]
Where \( dq_t \) is a Poisson process, which represents the mean value of jumps in a unit time, and the jumps- \( J \) follows an independent logarithm normal process. \( r_t \) return can be represented by:
\[
\ln(P_t / P_{t-1}) = \mu + \delta z + \sum_{i=1}^{\infty} \ln J_i
\]
(2.4)

Where \( n_t \) is the frequencies at which the jumps occur.

We could simulate the kurtosis property with a GARCH(1, 1) process. And its conditional variance is supposed to be the non-stochastic function of the squared disturbance item. \( n_t \) is a Poisson random variable with conditional jump intensity \( \lambda_t \); \( \theta \) is the mean of \( J_t \) in the interval of \( t-1 \) to \( t \), so that we could decomposed aggregate stochastic innovation \( \epsilon_t \) into a normal diffusion component \( \epsilon_u \) and a jump component \( \epsilon_h \). Then:
\[
\ln(P_t / P_{t-1}) = \mu + \delta z + \sum_{i=1}^{\infty} \ln J_i
\]
(2.5)

Where \( z_i \sim NID(0, 1); J_i \sim NID(\theta, \delta^2); h_t \) is the conditional heterogeneous auto-regression item. \( \epsilon_u \) is a GARCH(1, 1) process and \( \epsilon_h \) is a compounded Poisson process whose conditional density- \( \lambda_t \) is dependent of lagged value of itself - \( \lambda_{t-1} \). And the residual of the jump density is:
\[
\xi_{t-1} = E[n_{t-1} | h_t] - \lambda_{t-1} = \sum_{j=0}^{\infty} jP(n_{t-1} = j | h_t) - \lambda_{t-1}
\]
(2.6)

And the jump process could be expressed as an ARMA(1, 1) process as follows:
\[
\lambda_t = \lambda_0 + \rho \lambda_{t-1} + \phi_1 \xi_{t-1} + \phi_2 \epsilon_{t-1}
\]
(2.7)

where \( \xi_{t-1} \) is the lagged value of the exchange rate.

In order to study the leverage effect, clustering and consistent volatility, we decompose \( h_t \) as follows:
\[
h_t = \alpha_0 + \alpha_1 (\xi_{t-1} + \phi \epsilon_{t-1})^2 + \beta h_{t-1}
\]
(2.8)

\( \phi \) is for the study on leverage effect, and when positive shock arrives, \( \phi < 0 \), or otherwise, \( \phi > 0 \). The restrictive conditions are: \( \alpha_0 \geq 0, \alpha_1 \geq 0, 1 > \beta \geq 0 \), which is to guarantee a non-negative and stationary \( h_t \).

3. Data, Estimation Methodology and Results

A study of Nelson’s (1990) show that most of the GARCH process will converge to a diffusion process with continuous time while the interval of time approaches the infinitesimal. The property drive
us to use MLE to get the estimation of equation (2.8). The data for this paper come from TaiYang security service (220.168.18.34:8001), and the form of the data are respectively DowJones stock index and ShangHai security index 5-minute trading data. The time period is from Mar.7, 2005 to Dec.30.2005, and the observations are respectively 6872 and 6639, in which we eliminate the auction price in opening and closing stage.

The algorithms we use to estimate the parameters of equation (2.8) are from MLE and GARCH toolbox of the Matlab 7.0, and we adjust the core distribution likelihood functions to apply to the requirement of this paper. The adjusted logarithm maximum likelihood function is below:

$$L(\Theta; r_1, r_2, \ldots, r_T) = \sum_{t=2}^{T} \ln[f(r_t|\theta_1)]$$

Where

$$f(z_t|\theta_1, I_{t-1}) = \frac{1}{\sqrt{2\pi(h_t + j\delta^2)}} \exp\left(-\frac{(z_t/\sqrt{h_t + (\theta^2 + \delta^2)\lambda_t} + \theta\lambda_t - \theta)^2}{2(h_t + j\delta^2)}\right)$$

(3.1)

The result of the total properties test is given below:

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean value</th>
<th>S.D</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB value</th>
<th>KS value</th>
<th>ARCH value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHI</td>
<td>-0.00020</td>
<td>0.01549</td>
<td>1.212510</td>
<td>7.04027</td>
<td>138.7785</td>
<td>0.4847</td>
<td>122.266</td>
</tr>
<tr>
<td>DJI</td>
<td>-0.00013</td>
<td>0.00702</td>
<td>-0.257238</td>
<td>3.95295</td>
<td>5.960817</td>
<td>0.4925</td>
<td>88.2901</td>
</tr>
</tbody>
</table>

Both of the mean values of the returns of two markets are negative, and higher standard deviation for SHI could be observed than for DJI, which indicates that SH market have more abnormal volatility than has the US market. We also note that positive skewness for SHI relative to negative one for DJI, which verifies the conclusions on theory of Aggarwal and Inclan (1999) that augmented emerging stock markets has positive skewness compared with negative ones in western mature markets. The normality test-JB value shows that DJI for the behavior of the western mature markets has stronger effect than the emerging markets do. The ARCH value indicates that both markets have time-varying effect in heterogeneous variance into which we will research.

The variance decomposition and diffusive properties are given below:

<table>
<thead>
<tr>
<th>Index</th>
<th>Total variance</th>
<th>Weight of jump</th>
<th>Weight of diffusion</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\beta$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHI</td>
<td>0.0002402</td>
<td>69.74%</td>
<td>30.26%</td>
<td>0.0063</td>
<td>0.2001</td>
<td>0.9871</td>
<td>-0.3125</td>
</tr>
<tr>
<td>DJI</td>
<td>0.0000494</td>
<td>68.01%</td>
<td>31.99%</td>
<td>0.0022</td>
<td>0.0442</td>
<td>0.9058</td>
<td>-0.9907</td>
</tr>
</tbody>
</table>

In Table 2, we could observe that both markets have nearly 70% weight of jump volatility in the total variance, so we conclude that it is necessary to integrate jump variance into the diffusion process to elicit a volatility model with better explanatory power for the stock return. The $\alpha_1$ of the both markets are very significant with respect to their t-test, which shows that there exists time-varying effect in the volatilities process in both of the two markets. But nearly four times of the value of $\alpha_1$ with respect to the SHI compared with one of the DJI indicates that volatility of SHI is more sensitive to the time than
is DJI. The values of $\beta$ with respect to both of the indexes are positive, which means there exists clustering volatility in the conditional variance of the both series, while the value of $\beta$ of SHI is nearer to 1, which shows SHI has more consistent volatility. $\phi$ is used to illustrate the asymmetric leverage effect in the return series. The estimation result of the $\phi$ shows both markets have the leverage effect, while the effect of DJI is obviously stronger than that of the SHI. It indicates that the volatilities of the SHI are mainly driven by non-systematic volatilities compared with those of the DJI which are mainly driven by systematic volatilities.

As for the other component $\varepsilon_t$, the estimation result of its parameter is given as follows:

<table>
<thead>
<tr>
<th>Index</th>
<th>$\lambda$</th>
<th>$\theta$</th>
<th>$\delta$</th>
<th>$\rho$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\ln L$</th>
<th>$LR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHI</td>
<td>0.0027</td>
<td>-0.0011</td>
<td>0.0108</td>
<td>0.9798</td>
<td>0.1093</td>
<td>0.4378</td>
<td>5688.19</td>
<td>0.9543</td>
</tr>
<tr>
<td></td>
<td>(0.0405)</td>
<td>(0.2785)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0009)</td>
<td>(0.1307)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DJI</td>
<td>0.0335</td>
<td>-0.01 28</td>
<td>0.0058</td>
<td>0.5737</td>
<td>0.4706</td>
<td>0.0197</td>
<td>5102.30</td>
<td>0.9765</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0201)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\lambda_0$ represents the non-time-varying part of the jump density, while $\theta$ and $\delta$ are respectively the mean value and the standard deviation of the jump density. Comparing the two indexes, we find that the value of $\lambda_0$ of SHI is nearly 12 times of that for DJI, but with respect to the standard deviation, the value of $\delta$ for SHI is half of that for DJI. It could be explained that the jump process of the SHI is mainly caused by bigger volatility scope compared with that of DJI mainly caused by higher frequency. The fact that there is not a short selling mechanism in the Chinese security market which can be used to hedge mutual risk in the western markets is the reason why monotonous investment strategies could cause more irrational behaviors in security markets than diversified strategies in mature markets.

Time-varying coefficient $\rho$ is used to uncover the clustering and consistent properties of jump density $\lambda_t$. It could be observed that both of the two indexes have significant ARCH effects. But the value of $\rho$ for SHI approximates 1 compared with smaller one for DJI, which proves that systematic shocks and exogenous disturbances as “herding effect”, for example—have more consistent impact on irrational behaviors in the Chinese markets than in western markets. The value of $\gamma_1$ of SHI is far smaller than that of DJI, as indicates that SHI series is rather sensitive to the lagged jump and the leverage effect than to the arrival frequency of jump with respect to DJI series. So we conclude that the volatility of SHI series is more predictable than that of the DJI.

The estimations for $\gamma_2$ of both indexes are both positive, while the value of $\gamma_2$ of SHI is obviously smaller than that of the DJI, but $Z$—test for parameters shows that DJI with small $\gamma_2$ is more significant for parameter than SHI is, and it indicates that the impact of fluctuation of exchange rate on the jump process give better explanation to the jump of DJI series than to SHI series. It might be interpreted with our country’s mechanism of exchange rate which is marked to the US Dollar.

4. Conclusion

Based on the mixed jump-GARCH model, this paper empirically compares the volatility of the SHI series with that of the DJI. Our model allows the jump intensity to be autoregressive and dependent on the lagged exchange-rate changes, and then it can examine the stylized features of the index returns in our country’s markets and capture several distinguishing features of them. As expected, the volatility is higher in Chinese markets as compared to the securities markets in the US. The high volatilities are accompanied by negative returns even after the shift of the exchange-rate mechanism. The leverage effect of normal return shocks in the our country’s markets is less than in the US, but the asymmetric
response of the volatility to jump-like shocks is greater in our country’s markets.

References