Valuation Model of Growth Companies Based on Real Option and Its Empirical Investigation

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Abstract: Discounted cash flow is the main tool for valuing projects and companies, and Real options techniques can augment valuation. We apply real option theory and capital-budgeting techniques to the problem of valuing a growth company. This paper simulates the case company’s value based on a revised Schwartz and Moon valuation model, and at last analyzes key drivers of improving its value. We formulate the simplified model in continuous time, estimate its parameters, and solve the model by an empirical investigation. Perhaps more importantly, this paper offers insights into the practical implementation the model in our country. An important challenge to implementing the original model was estimating the various parameters. Here, this paper improve the procedure by setting the speed of adjustment parameters equal to one another, and by inferring the risk-adjustment parameter from the observed beta of the company's stock price.

Keywords: Company valuation, Empirical study, Real options, Uncertainty

1 Introduction

The company is the special merchandise of the capital market. Its value and market price, reflecting the overall profitability of the company, are core problems of each contractual body of company. So maximizing the company value creation becomes a strategic ultimate target of company. The company value and valuation becomes the important tool of the company management. Company valuation is a process of measuring company’s value under a certain assumptions with financial theories, models, methodologies, etc.

Company valuation not only provides a scientific and reasonable measure of value of external property exchange such as M&A and IPO, but also is an important tool for company value management. Because of the relatively late start of the assets appraisal industry in our country and the weak research of the company valuation theory, the problem of misusing company valuation methods has existed in our country for a long time. Through analysis of large amount of literature and many years’ company valuation practice; I have the opinion that a lot of problems in the theory and methods of company valuation demand prompt solution.

The company value and the ultimate target of the financial management is the maximization of the company value. Therefore, cash flow, as the basic of the evaluation, should be the overall cash flow occurring in all the company business activities instead of the operating cash flow, free cash flow, equity cash flow or debt cash flow. At present there are many estimating models, including time-series model, stochastic series model, fuzzy estimating model, multi-variable regression model, gray model and hierarchy analysis model, and they are some help to estimating the overall cash flow. We have done systematical research on the theory and methods of company valuation by means of using comparative research and empirical research. We methodically explain and meticulously compare the six current methods of company valuation. It can be concluded form the researches that though the assets based valuation method and the real option valuation method have certain scope of application, they don’t have universality; the discounted income valuation method is the popular method of valuation, but it still needs theoretically clarifying and improving in long-term return forecast; the relative comparison valuation method is easy to use, however, because of the monotonous factor the model considers and the high degree of subjectivity in discrepancy adjustment, it makes standard measurement necessary.

This paper applies risk-neutral thinking of real option theory into traditional Discounted Cash Flow method, in order to discuss how to evaluate companies under uncertainty. In the process of valuation, this paper uses the revised Schwartz and Moon\(^{[1]}\) valuation model to simulate future cash flows of eBay,
and describes uncertainties it faced with stochastic processes. Additionally and at last analyzes key drivers of improving its value.

2 Literature Review

Real options represent the most recent capital budgeting techniques to value growth opportunities. In the Schwartz-Moon model, the main source of option value is the right to walk away from an unprofitable operation. Schwartz and Moon concluded that the prices of the companies observed at the time of their studies far exceeded the estimated real options values of the companies, which in turn significantly exceeded DCF values. But in their model, once investment is completed, the owner of the project receives the value of the approved drug in the form of a single cash flow. However, some of the same issues discussed in this paper are important differences, both in the formulation of the problem and in the solution procedure between Schwartz and Moon and this paper.

Unobserved component or factor models are popular in the finance literature\cite{2,3,4}. In the option pricing literature, Bates\cite{5} and Taylor and Xu\cite{6} investigate two-factor stochastic volatility models. Duffie, Pan, and Singleton\cite{7} provide a general continuous-time frame work for the valuation of contingent claims using multi factor affine models. Eraker\cite{8} suggests the usefulness of a multi factor approach based on his empirical results. Alizadeh, Brandt, and Diebold\cite{9} uncover two factors in stochastic volatility models of exchange rates using range-based estimation. Bollerslev and Zhou\cite{10}, Brandt and Jones\cite{11}, Chacko and Viceira\cite{12}, Chernov, Gallant, Ghysels, and Tauchen\cite{13}, and Maheu\cite{14} also find that two-factor stochastic volatility models out perform single factor models when modeling daily asset return volatility. Adrian and Rosenberg\cite{15} investigate the relevance of a two-component volatility model for pricing the cross section of stock returns. Unobserved component models are also popular in the term structure literature; although in this literature the models are more commonly referred to as multifactor models. See for example Dai and Singleton\cite{16}, Duffee\cite{17}, Duffie and Singleton\cite{18}, and Pearson and Sun\cite{19}. Interesting parallels exist between our approach and results and stylized facts in the term structure literature. In the term structure literature, short-run fluctuations are customarily modeled around a time-varying long-run mean of the short rate. In our frame work we model short-run fluctuations around a time-varying long-run volatility.

In Schwartz and Moon\cite{20} they developed a model for pricing Internet companies using real options theory and modern capital budgeting techniques. The novelty of the approach is that uncertainty about the key variables which determine the value of an Internet company play a central role in the valuation. In particular, that paper considers uncertainty in both the revenues and the rates of growth in revenues. Schwartz confronts this issue head-on in his usual dynamic fashion. In a paper coauthored with Mark Moon, Schwartz focuses on the impact of cash flow uncertainty on the value of internet companies and recognizes that the real options approach, which has proved so useful in modern capital budgeting, can be effective in pricing companies with multiple sources of volatility in their business models. An extension of earlier work which appeared in the Financial Analysts Journal (Schwartz and Moon, 2000) adds a third source of uncertainty, variable cost volatility, to the initial mix of uncertain revenue and volatile revenue growth. The extended model also incorporates cash flows from the purchase and depreciation of physical assets and uses market data to infer the risk premium appropriate for the discounting process. These enhancements create a generalized asset pricing approach that can be applied to any company or industry where youth and/or technological change create multiple interacting sources of uncertainty. Despite its many improvements, the extended Schwartz and Moon model still projects a share price almost 50% above the value of a sample stock in the spring of 2001. This discrepancy proves that we still do not fully understand the components of value for companies in emerging industries. Much space on the asset pricing palette is left open for the further blending of real options theory and market data to forecast prices for other internet companies, or for other in a state of flux, using discounted cash flow techniques.

Schwartz and Moon’s model is one of the most practical in all real options pricing models. But due to the different accountant system and data obtaining problem, their models have some trouble when using in our country. In this paper we first revise the Schwartz and Moon’s model in the following two aspects:
partition of cost and estimation of parameters. Then we make use of this revised model to evaluate a listed company eBay. This paper implies real option theory brings forward a new systematic way of thinking for company valuation, but real option pricing method still cannot be a precise measure of company valuation, because of complexities of its methodologies and technologies.

3 Valuation method base on Schwartz and Moon model

3.1 Revenues
Consider an Internet company with an instantaneous rate of revenues (or sales) at time \( t \) given by \( R_t \), there are three basic sources of uncertainty in the valuation of Amazon. First, revenues are uncertain. Second, the growth of revenues is uncertain. Third, variable costs are uncertain. But we can’t just say that those three quantities are uncertain, without specifying how they change over time. Assume that the dynamics of these revenues are given by the stochastic differential equation:

\[
\frac{dR_t}{R_t} = \mu_t dt + \sigma_t dw^t_t
\]  

(3.1)

The growth rate \( \mu_t \) isn’t deterministic and constant over time, \( \sigma_t \) is volatility in the rate of revenue growth, and \( w^t_t \) is a random variable that reflects the draw from a normal distribution ( \( N(0,1) \) ), and is independent over time. However, this is not a realistic assumption for the revenues of growth companies. It is safe to say that the growth of those companies, although uncertain, will decrease in volatility over time, as will the revenues themselves.

First, we will assume that the volatility of revenues is decreasing deterministically over time, or

\[
\sigma_{t+1} = (1-\phi)\sigma + \phi \sigma_t, 0 < \phi < 1
\]  

(3.2)

In the above expression, \( \sigma_{t+1} \) will converge to a steady state value of \( \sigma \) as time \( t \) progresses, or \( t \rightarrow \infty \). The rate of convergence will depend on the parameter \( \phi(0 < \phi < 1) \). Therefore, the above equation captures the fact that volatility of revenues will decrease with time until they reach a value of \( \sigma \).

3.2 Growth of Revenues
There also assume that the growth rate of revenues \( \mu_t \) is uncertain and follows the process:

\[
\mu_t = (1-\phi)\bar{\mu} + \phi \mu_{t-1} + \eta_t w^2_{t}
\]  

(3.3)

Where \( w^2_{t} \) has a standard normal distribution ( \( N(0,1) \) ) and is independent over time. The volatility of the growth rate of revenues is also a deterministic function of time, and dwindles to zero, as \( t \rightarrow \infty \), or:

\[
\mu_{t+1} = \phi \mu_{t}
\]  

(3.4)

In other words, as time passes, the growth rate will be less and less variable until it reaches levels of stability observed in other “mature” companies. Therefore, over time, we should observe the growth rate of revenues \( \mu_t \) to converge \( \bar{\mu} \):

Note that, from the above discussions, we have established that

\[
\sigma_t \rightarrow \bar{\sigma}, as \ t \rightarrow \infty ; \ \mu_t \rightarrow \bar{\mu}, as \ t \rightarrow \infty
\]

Therefore, returning back to equation (1), we can say that:

\[
\frac{dR_t}{R_t} = \bar{\mu} dt + \bar{\sigma} dw^t_t, as \ t \rightarrow \infty
\]  

(3.5)

or that in the long run, the revenues of a company will grow at a steady rate of \( \bar{\mu} \) and will have a steady volatility \( \bar{\sigma} \).

3.3 Variable Cost
Lastly, we assume that production cost changes over time. In general, production cost can be decomposed into variable cost and fixed cost. Moreover, variable cost is usually a fraction of revenues. Therefore, we may be tempted to write an equation for cost as

$$C_t = F + \gamma_t R_t$$  \hspace{1cm} (3.6)

Where $\gamma$ is a fraction between 0 and 1 and $F$ represent all fixed cost. However, such an equation makes an implicit assumption that you are asked to discuss in your Project. Instead, we model cost as:

$$C_t = (1 - \phi)\gamma_t + \phi_{t-1} + \phi_t w_t^3$$  \hspace{1cm} (3.7)

Where $w_t^3$ has a standard normal distribution ($N(0,1)$) and is independent over time. Note that the process $\gamma_t$ has the same “flavor” as the growth rate of revenues, $\mu_t$, described above. Indeed, let the volatility $\phi_t$ of $\gamma_t$ be $\phi_t = (1 - \phi)\bar{\phi} + \phi_{t-1}$. From the above discussion, we should know that, as $t \to \infty$, and have $\phi_t \to \bar{\phi}$.

### 3.4 Rates of Convergence

So far, we have defined the three random processes $R_t, \mu_t, \gamma_t$. Note that they all depend on a very crucial parameter $\phi$, which determines how fast those processes converge to their long-horizon values. Schwartz and Moon propose a way of calibrating the parameter $\phi$ by asking the question: “How long will it take for the effect $\phi$ to decrease by half?” This is called the “half-life” of a process. We find the half-life of a process by letting:

$$k^k = 1/2$$  \hspace{1cm} (3.8)

Taking natural log, we can say that $k \ln(\phi) = \ln(1/2)$, or

$$k = \frac{\ln(1/2)}{\ln(\phi)} \approx -0.69$$  \hspace{1cm} (3.9)

If we specify $\phi$, we have in fact specified the behavior of all processes. For example, if $\phi = 0.5$, then $k = \ln(0.5) / \ln(0.5) = 1$, or it takes one year for all of the above processes to reach half of their long-term equilibrium. If $\phi = 0.9$; then $k = \ln(0.5) / \ln(0.9) = 6.58$, or it will take approximately 6.58 years for all processes to reach half of their long-term equilibrium. In other words, by specifying $k$ -the half-life of the processes- we are incorporating into the model how fast $R_t, \mu_t, \gamma_t$, and all the variances converge to their long-run values.

### 3.5 Rest of the Model

Thus far, we have defined the random variables of our model. Now, we will put the rest of the model together. Assume that the corporate tax rate is $\tau_C$ and that Amazon’s investment (hardware, software, building, etc.) depreciates at rate $\text{Dep}$. Then, the after-tax net income of Amazon is:

$$Y_t = (R_t - C_t - \text{Dep})(1 - \tau_C)$$  \hspace{1cm} (3.10)

We probably remember from our accounting class that corporate taxes are paid only if there is no loss-carry-forward, i.e. if there are no losses in previous periods that offset current gains. Schwartz and Moon allow for such a possibility in their model, but we will abstract from it, in the interest of simplicity and brevity. However, the distributed program incorporates those details. Denote by $\text{Capx}_t$ the capital expenditures that Amazon has in period $t$. Therefore, we can define the amount of cash available to Amazon in any given period by $X_t$, where

$$X_t = (1 + \gamma_t)X_{t-1} + Y_{t-1} + \text{Dep}_t - \text{Capx}_t$$  \hspace{1cm} (3.11)

Schwartz and Moon include the untaxed interest earned on the cash available in the dynamics to make the valuation results insensitive to when the cash flows are distributed to the security-holders. Schwartz and Moon assume that Amazon will go bankrupt when its available cash reaches a
predetermined negative number, say $X^\ast$. The threshold $X^\ast$ is allowed to be negative to accommodate the possibility of future financing.

The ultimate objective of this model is to determine the value of a company today. Schwartz and Moon show that the payoff from Amazon from today to $T$ periods ahead in the future, under the risk-neutral valuation, is $X_T + M \times (R_T - C_T)$, where $M$ is a multiple. Therefore, the value of the company today until horizon $T$ is:

$$ V_t = E \left[ X_T + M \times (R_T - C_T)e^{-\gamma T} \right] $$

(3.12)

Note that the expectation is taken with respect to the risk-neutral probabilities. Here the mathematics is a bit more complicated, but the idea is exactly the same.

We have found the rational market value of a company! In order to determine the price of a share of the company, we need to know the capital structure of the company in detail, i.e. how many shares are outstanding, how many shares are likely to be issued to stock option holders, and convertible bondholders. We also need to know how much of the cash flows will be available to the shareholders after coupon (interest) and principal payment to the bondholders. Schwartz and Moon make some minor assumptions in order to tackle those complications, but none of them are essential enough to require our attention.

After writing a few more formulas, we can finally express the stock price of a company, $S_t$, as a function of the initial variables, $R_T$, $\mu_t$, $\gamma_t$, and $X_t$ or $t$,

$$ S = \text{Function}(R_t, \mu_t, \gamma_t, X_t, t) $$

(3.13)

Characterizing the behavior of this function involves Itô’s Lemma and is beyond the scope of this class. Moreover, the above problem is too complicated to be solvable by hand. Schwartz and Moon use simulations in order to find a solution of the entire value of a company and its price per share.

$$ dV = V_R dR + V_\mu d\mu + V_\gamma d\gamma + V_X dX + V_t dt + \frac{1}{2} V_{RR} dR^2 + \frac{1}{2} V_{\mu\mu} d\mu^2 + V_{R\mu} dRd\mu $$

(3.14)

Applying Itô’s lemma to this expression, we can obtain the dynamics of the value of the company as (3.14). The volatility of the company’s value can be derived directly from

$$ \sigma^2 = \frac{1}{dt} \text{var} \left( \frac{dV}{V} \right) = \left( \frac{V_R}{V} \sigma R \right)^2 + \left( \frac{V_\mu}{V} \sigma \mu \right)^2 + 2 \frac{V_R V_\mu}{V^2} \sigma R \sigma \rho $$

(3.15)

The model can then be used to determine not only the value of the company but also its volatility.

4 Empirical Investigations

4.1 Methodology

The Schwartz-Moon model involves specifying a rather large set of input parameters. Some of them are directly observable, such as the initial revenues, the initial cash balance, or the risk-free rate of interest, whereas others, such as the initial volatility of the expected rate of growth in revenues or the mean-reversion speed parameters, are not directly observable and have therefore to be estimated. This would be analogous to estimating an implied volatility within the standard model, with the difference being that instead of a scalar parameter, a vector of parameters had to be estimated. As an implicit determination of all parameters is not realistic, we must identify a smaller subset which makes the minimization problem reasonable. Analogous to the standard model, one can expect volatility parameters to have the largest impact on option prices. The main stochastic variable which drives the firm value is the amount of revenues. Hence we choose the respective volatility function $\sigma_t$ to be estimated implicitly. This function is determined by the initial level $\sigma_0$, the long-term mean $\overline{\sigma}$, and the speed of mean-reversion $k_\sigma$, according to (3.2):

$$ \sigma_t = \sigma_0 e^{-k_\sigma t} + \overline{\sigma} (1 - e^{-k_\sigma t}) $$

(3.16)

However, even within these three parameters there are still large trade-off effects; thus we fix the
long-term mean and only estimate the initial level \( \sigma_0 \) and the speed of mean reversion \( k_\sigma \). Furthermore, we need one parameter to let the model reflect the observed stock price. Given the level of revenues, the variable which has the largest impact on the firm value and thus the stock price, is the growth rate function. So we choose the initial growth rate of revenues \( \mu_0 \) to be determined implicitly in order to let the model-implied firm value be consistent with the observed stock price. Accordingly, there are a total of three parameters to be implicitly estimated. The target function we seek to minimize is the relative mean squared error (RMSE) of the model implied option prices with respect to the observed option prices. To avoid problems with the relativity of the error, which can arise with very low prices for out-of-the-money options on the one hand and very high prices with in-the-money options on the other, we consider straddles, i.e., combinations of calls and puts with the same strike price, instead of single calls and puts. Summing up, we solve the minimization problem.

\[
(\mu_0^*, \sigma_0^*, k_\sigma^*) = \arg \min \frac{1}{n} \sum_{i=1}^{n} \left( \frac{c_i - c_i^{SM} + p_i - p_i^{SM}}{c_i + p_i} \right)
\]

(3.17)

\[
S^{SM}(\mu_0, \sigma_0, k_\sigma) = S_0
\]

(3.18)

Where \( c_i, p_i \) and \( S_0 \) denote the observed prices for the \( i \)-th call, the \( i \)-th put, and the stock, respectively, and \( c_i^{SM}(\mu_0, \sigma_0, k_\sigma) \), \( p_i^{SM}(\mu_0, \sigma_0, k_\sigma) \) and \( S^{SM} \), the respective model-implied values. To obtain the remaining firm-specific parameters used for the valuation we apply different approaches. Whereas the initial cash balance \( X_0 \) and the initial property, plant, and equipment \( PPE_0 \) are explicitly stated in the companies’ quarterly reports (10-K form), all other parameters must be estimated and are either inferred from historical data or based upon subjective appraisals, which are stated below. For the initial annual revenues \( R_0 \), we use the product of four times the actual quarterly revenues, where we infer \( R_0 \) from the last four quarters to account for seasonal fluctuations. The initial volatility \( \eta_0 \) of the expected growth rate is set to 20% per year. The initial variable cost rate \( \gamma_0 \) and the fixed costs \( F \) are obtained by regressing all cash costs except cost of revenues and stock-based compensation of the last eight quarters onto the revenues. The resulting axis intercept is used as an estimate for the fixed costs \( F \); the slope plus the average of the last eight quarterly relative costs of revenues is used as an estimate for the initial variable cost rate \( \gamma_0 \). For the long-term average \( \gamma \), we use the last value of \( \gamma_0 \) in our data series, as this value has already become quite stable. The initial volatility \( \varphi_0 \) of the variable cost rate is also inferred from the last eight quarters. Furthermore, we use \( \varphi_0 \) as the best estimate for the long-term average \( \varphi \). The long-term expected rate of growth in revenues \( \bar{\mu} \) is set to 5% per year and the long-term volatility \( \bar{\sigma} \) of revenues is set to 10% per year. In specifying the mean-reversion speed parameters \( k_\mu, k_\eta \) and \( k_\gamma \), we follow Keiber et al.\(^{21}\), who suggest values of 0.4. The estimates for the rate of investment \( CR \) and the depreciation rate \( DR \) are based on the respective average relations of the last eight quarters. The risk premiums \( \lambda_R, \lambda_\mu, \lambda_\gamma \) are defined in line with the assumptions leading to the Intertemporal Capital Asset Pricing Model as the product of the volatility \( \sigma_M \) of the market portfolio and the correlation \( \rho_{0,M} \) between the respective process and the return on the market portfolio. Estimates for \( \sigma_M \) and \( \rho_{0,M} \) are derived from the last eight quarters using the Dow Jones Industrial Average as a substitute for the market portfolio. To simplify matters, we assume that the stochastic processes are not correlated. Furthermore, following Schwartz and Moon we negate the existence of any future financing possibilities; thus \( X^* = 0 \) holds for all companies. The EBITDA-multiple \( M \) is taken, as frequently done by practitioners, to be 10. The corporate tax rate \( \tau \) is set to 35%. The risk-free rate of interest is inferred from the U.S. treasury yield curve, using the yield to maturity for 10 years at the day of announcement of the respective quarterly
results. The horizon $T_h$ for the valuation is 25 years. We assume that all in-the-money stock options will be exercised at their expiry dates. The receipts from the exercise price increase the future cash balance $X_T$. We use the diluted number of shares as reported in the quarterly reports for eBay, and use market data from January 2005 through June 2008. We apply closing quotes and use the average of bid and ask quotes. Calculations are performed on a weekly basis, running 40,000 simulations in each case.

4.2 Parameter Estimation Results
We first examine the estimated parameters. The calculations show that the RMSE is rather insensitive with respect to the volatility mean-reversion speed parameter $\sigma_k$ as long as the parameter is within a certain range of about 0.0 to 0.2. Hence we choose to calibrate the model with a fixed $\sigma_k$ for all estimation dates to render the remaining parameters more comparable. The best fit is achieved with $\sigma_k = 0.5$

![Image of Figure 1](image.png)

**Figure 1 Mean implied and historical initial volatility of growth rate in revenues estimated after the announcement of the respective quarterly results**

Considering the initial volatility of the rate of growth in revenues, $\sigma_0$, Figure 1 compares the quarterly mean of the implied volatilities, calculated on a weekly basis after announcement of the quarterly results for the respective quarter, with the historical volatilities inferred from the last eight quarters, which is the standard procedure for the estimation of this parameter. We find a discrepancy between the historical and the implied volatilities, in the direction that the latter is always higher than the corresponding historical estimate. The comparatively high implied volatility can be explained by the fact that within the model, the initial volatility $\sigma_0$ of the rate of growth in revenues must be responsible for all stock price fluctuations. Real stock price variations, which are reflected in option prices, might stem from a number of different sources, e.g., stochastic interest rates, changes in the market risk premium, or variations in the market expectation of long-term average parameters such as $\bar{\mu}$ etc., which are not incorporated in the model. These other sources of uncertainty are completely mapped onto the implicitly estimated model parameter $\sigma_0$, which must thus be larger than the value induced by an exogenous estimation.

Regarding the initial expected rate of growth in revenues, $\mu_0$, the standard procedure would be to base the estimation of this parameter on historical data, i.e., the average realized growth rate in revenues
during the last quarters. Figure 2 compares this historical estimate, based on four quarters, with the quarterly mean implied initial expected rate of growth after announcement of the quarterly results for the respective quarter. We observe considerable differences for eBay, where implied growth rates are nearly always lower than the corresponding historical values.

![Figure 2](image)

**Figure 2 Mean implied and historical initial expected rate of growth $\mu_0$ in revenues estimated after the announcement of the respective quarterly results**

## 5 Conclusion

In this paper we argue that DCF and risk-neutral valuation both play an essential role in capital budgeting calculations. Traditional DCF is a necessary input for a real option valuation, and real option methods can greatly simplify otherwise daunting valuation problems. Even with valuation problems that seem relatively straightforward, option valuation methods are helpful. But rather than discuss valuation methods using the term “real options”, it may be more productive to concentrate on the details of the economic and valuation issues that arise in specific contexts and then decide the best way to obtain accurate prices.

Despite survey evidence reporting that most managers do not claim to use real options methods when making capital budgeting decisions, academic studies generally find both managerial behavior and market pricing to be consistent with the predictions of real option models. Managers can be expected to find rules of thumb and ad hoc modifications of DCF and other traditional valuation approaches that lead to better evaluation of a given company.

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